We calculate the intershell resistance $R_{21}$ in a multiwall carbon nanotube as a function of temperature $T$ and Fermi level $\epsilon(F)$ (e.g., a gate voltage), varying the chirality of the inner and outer tubes. This is done in a so-called Coulomb drag setup, where a current $I_1$ in one shell induces a voltage drop $V_2$ in another shell by the screened Coulomb interaction between the shells neglecting the intershell tunneling. We provide benchmark results for $R_{21} = V_2/I_1$ within the Fermi liquid theory using Boltzmann equations. The band structure gives rise to strongly chirality-dependent suppression effects for the Coulomb drag between different tubes due to selection rules combined with mismatching of wave vector and crystal angular momentum conservation near the Fermi level. This gives rise to orders of magnitude changes in $R_{21}$ and even the sign of $R_{21}$ can change depending on the chirality of the inner and outer tube and misalignment of inner and outer tube Fermi levels. However for any tube combination, we predict a dip (or peak) in $R_{21}$ as a function of gate voltage, since $R_{21}$ vanishes at the electron-hole symmetry point. As a by-product, we classified all metallic tubes into either zigzaglike or armchairlike, which have two different nonzero crystal angular momenta $m(a)$, $M(b)$ and only zero angular momentum, respectively.