A frame in a Hilbert space $H$ is a countable collection of elements in $H$ that allows each $f \in H$ to be expanded as an (infinite) linear combination of the frame elements. Frames generalize the well-known orthonormal bases, but provide much more flexibility and can often be constructed with properties that are not possible for orthonormal bases. We will present the basic facts in frame theory with focus on their operator theoretical characterizations and discuss open problems concerning representations of frames in terms of iterations of a fixed operator. These problems come up in the context of dynamical sampling, a topic that has recently attracted considerable interest within harmonic analysis. The goal of the paper is twofold, namely, that experts in operator theory will explore the potential of frames, and that frame theory will benefit from insight provided by the operator theory community.