Fractional and complex pseudo-splines and the construction of Parseval frames

Pseudo-splines of integer order \((m, ℓ)\) were introduced by Daubechies, Han, Ron, and Shen as a family which allows interpolation between the classical B-splines and the Daubechies’ scaling functions. The purpose of this paper is to generalize the pseudo-splines to fractional and complex orders \((z, ℓ)\) with \(α = \Re z ≥ 1\). This allows increased flexibility in regard to smoothness: instead of working with a discrete family of functions from \(C^m, m∈N_0\), one uses a continuous family of functions belonging to the Hölder spaces \(C^{α−1}\). The presence of the imaginary part of \(z\) allows for direct utilization in complex transform techniques for signal and image analyses. We also show that in analogue to the integer case, the generalized pseudo-splines lead to constructions of Parseval wavelet frames via the unitary extension principle. The regularity and approximation order of this new class of generalized splines is also discussed.