Eigenfrequencies and eigenmodes of a beam with periodically continuously varying spatial properties

A beam with periodically continuously varying spatial properties is analyzed. This structure is a generic model for various systems widely used in industry, e.g. risers, rotor blades, and similar. The aim is to reveal effects of periodic spatial modulation both on the beam eigenfrequencies and eigenmodes. Special attention is given to "mid-frequency" eigenmodes having period of the same order as the period of modulation, which cannot be captured by the conventional analytical methods. In particular, the paper addresses prediction of bandgaps and their influence on the distribution of eigenfrequencies. For analyzing the problem considered, the method of varying amplitudes is employed. A connection of this method with the classical Hill's infinite determinant method and the method of space-harmonics is noted. A dispersion relation of the considered non-uniform periodic structure is obtained, and values of the modulation amplitudes at which frequency bandgaps arise are determined. It is shown that eigenfrequencies of the beam can lie within the bandgaps, and that such eigenfrequencies can be considerably affected by modulation. It is revealed that there is an abrupt shift in the effect of modulation on eigenfrequencies, and that modulations of the beam mass per unit length and of the beam stiffness affect them oppositely. It is shown that eigenmodes having a period close to the period of modulation comprise a long-wave component; this illustrates the capacity of non-uniform structures to sustain long-wave oscillations on comparatively high frequencies.