Dynamics of Rigid Bodies and Flexible Beam Structures

Rotational motion is a frequently occurring aspect in many engineering applications such as automobiles, rotating machinery or wind turbine rotors. The present thesis is organized in four parts - all concerning development of efficient computational methods for modeling the dynamic behavior of rigid bodies and flexible beam structures with emphasis on the rotational motion.

The first part deals with motion in a rotating frame of reference. A novel approach where the equations of motion are formulated in a hybrid state-space in terms of local displacements and global velocities is presented. It is shown that particular simplifications are obtained when the same interpolation functions are used for both state-space variables, whereby all inertia effects can be represented via the classic constant mass matrix. The hybrid state-space constitutes the basis for developing a conservative time integration scheme and an associated algorithmic dissipation, that affects only the local motion.

In the second part a conservative integration scheme for rigid body motion in a global frame of reference is presented. A fully algebraic representation of the rotational motion is obtained by using either four quaternion parameters or nine convected base vector components. In both cases, the equations of motion are obtained via Hamilton’s equations by including the kinematic constraints associated with the redundant rotation description by means of Lagrange multipliers. A special feature of the formulation is that these can be replaced by a projection operator applied to the external potential gradient and possible external constraint gradients. The third part presents a novel two-node beam element capable of undergoing arbitrary large displacements and finite rotations. The element is expressed explicitly in terms of the global components of the position vectors and associated convected base vectors for the element nodes. The kinematics is expressed in a homogeneous quadratic form and the constitutive stiffness is derived from complementary energy of a set of equilibrium modes, each representing a state of constant internal force or moment. This approach avoids local interpolation of kinematic variables, which makes the formulation inherently locking-free and frame-invariant. In the final part of the thesis a multi-level optimization procedure for wind turbine blades is presented. Computational procedures similar to the ones presented in this thesis constitute the basis for a preliminary optimization using a multibody beam model of the full wind turbine. The novel aspects considered here consist of automatic generation of a 3D FE-model of a single blade for detailed stress and buckling analysis which cannot be performed on the beam model, and utilization of the results for updating the beam model for a subsequent iteration step. It is demonstrated that convergence between the two models is obtained in very few iterations.