Construction of smooth compactly supported windows generating dual pairs of Gabor frames

Let \( g \) be any real-valued, bounded and compactly supported function, whose integer-translates \((Tkg)_{k \in \mathbb{Z}}\) form a partition of unity. Based on a new construction of dual windows associated with Gabor frames generated by \( g \), we present a method to explicitly construct dual pairs of Gabor frames. This new method of construction is based on a family of polynomials which is closely related to the Daubechies polynomials, used in the construction of compactly supported wavelets. For any \( k \in \mathbb{N} \cup \{\infty\} \) we consider the Meyer scaling functions and use these to construct compactly supported windows \( g \in C^k(\mathbb{R}) \) associated with a family of smooth compactly supported dual windows. For any \( n \in \mathbb{N} \) the pair of dual windows \( g, h_n \in C^k(\mathbb{R}) \) have compact support in the interval \([-2/3, 2/3]\) and share the property of being constant on half the length of their support. We therefore obtain arbitrary smoothness of the dual pair of windows \( g, h_n \) without increasing their support.