We address several concerns related to the derivation of drift-ordered fluid equations. Starting from a fully Galilean invariant fluid system, we show how consistent sets of perturbative drift-fluid equations in the case of an isothermal collisionless fluid can be obtained. Treating all the dynamical fields on equal footing in the singular-drift expansion, we show under what conditions a set of perturbative equations can have a non-trivial quasi-neutral limit. We give a suitable perturbative setup where we provide the full set of perturbative equations for obtaining the first-order corrected fields and show that all the constants of motion are preserved at each order. With the dynamical field variables under perturbative control, we subsequently provide a quantitative analysis by means of numerical simulations. With direct access to first-order corrections, the convergence properties are addressed for different regimes of parameter space and the validity of the first-order approximation is discussed in the three settings: cold ions, hot ions, and finite charge density.