The simplest network of coupled phase-oscillators exhibiting chimera states is given by two populations with disparate intra-and inter-population coupling strengths. We explore the effects of heterogeneous coupling phase-lags between the two populations. Such heterogeneity arises naturally in various settings, for example, as an approximation to transmission delays, excitatory-inhibitory interactions, or as amplitude and phase responses of oscillators with electrical or mechanical coupling. We find that breaking the phase-lag symmetry results in a variety of states with uniform and non-uniform synchronization, including in-phase and anti-phase synchrony, full incoherence (splay state), chimera states with phase separation of 0 or π between populations, and states where both populations remain desynchronized. These desynchronized states exhibit stable, oscillatory, and even chaotic dynamics. Moreover, we identify the bifurcations through which chimeras emerge. Stable chimera states and desynchronized solutions, which do not arise for homogeneous phase-lag parameters, emerge as a result of competition between synchronized in-phase, anti-phase equilibria, and fully incoherent states when the phase-lags are near +/-π/2 (cosine coupling). These findings elucidate previous experimental results involving a network of mechanical oscillators and provide further insight into the breakdown of synchrony in biological systems. Published by AIP Publishing.