Adaptive Kronrod-Patterson integration of non-linear finite-element matrices

Efficient simulation of unsaturated moisture flow in porous media is of great importance in many engineering fields. The highly non-linear character of unsaturated flow typically gives sharp moving moisture fronts during wetting and drying of materials with strong local moisture permeability and capacity variations as result. It is shown that these strong variations conflict with the common preference for low-order numerical integration in finite element simulations of unsaturated moisture flow: inaccurate numerical integration leads to errors that are often far more important than errors from inappropriate discretization. In response, this article develops adaptive integration, based on nested Kronrod-Patterson-Gauss integration schemes: basically, the integration order is adapted to the locally observed grade of non-linearity. Adaptive integration is developed based on a standard infiltration problem, and it is demonstrated that serious reductions in the numbers of required integration points and discretization nodes can be obtained, thus significantly increasing computational efficiency. The multi-dimensional applicability is exemplified with two-dimensional wetting and drying applications. While developed for finite element unsaturated moisture transfer simulation, adaptive integration is similarly applicable for other non-linear problems and other discretization methods, and whereas perhaps outperformed by mesh-adaptive techniques, adaptive integration requires much less implementation and computation. Both techniques can moreover be easily combined. Copyright © 2009 John Wiley & Sons, Ltd.