A Two-Stage Decomposition of High School Timetabling applied to cases in Denmark

Integer Programming (IP) has been used to model educational timetabling problems since the very early days of Operations Research. It is well recognized that these IP models in general are hard to solve, and this area of research is dominated by heuristic solution approaches. In this paper a Two-Stage Decomposition of an IP model for a practical case of high school timetabling is shown. This particular timetabling problem consists of assigning lectures to both a timeslot and a classroom, which is modeled using a very large amount of binary variables. The decomposition splits this model into two separate problems (Stage I and Stage II) with far less variables. These two separate problems are solved in sequence, such that the solution for the Stage I model is given as input to the Stage II model, implying that irreversible decisions are made in Stage I. However, the objective of the Stage II model is partly incorporated in the Stage I model by exploiting that Stage II can be seen as a minimum weight maximum matching problem in a bipartite graph. This theoretically strengthens the decomposition in terms of global optimality. The approach relies on Hall’s theorem for the existence of matchings in bipartite graphs, which in its basic form yields an exponential amount of constraints in the Stage I model. However, it is shown that only a small subset of these constraints is needed, making the decomposition tractable in practice for IP solvers. To evaluate the decomposition, 100 real-life problem instances from the database of the high school ERP system Lectio are used. Computational results show that the decomposition performs significantly better than solving the original IP, in terms of both found solutions and bounds.