Robotic system and method for manufacturing of objects

The present disclosure relates to a method and a system for manufacturing a mould (17) for creation of complex objects, such as concrete objects, by controlling and moving two end effectors (1) of a robotic system, the two end effectors (1) having a flexible cutting element (3) attached to and extending between the two end effectors (1), the method comprising the steps of: defining at least one surface (8) representing the inner surface of the mould (17); dividing the surface (8) into a number of segments represented by planar curves (9, 11, 12) on the surface (8); for each planar curve, calculating at least one elastic curve representing the planar curve; for each calculated elastic curve, calculating a set of data corresponding to placement and direction of the two end effectors (1) for configuring the flexible cutting element to a shape corresponding to the calculated elastic curve; sequentially positioning the end effectors (1) according to each set of data.

Rationalization in architecture with surfaces foliated by elastic curves

We develop methods for rationalization of CAD surfaces using elastic curves, aiming at a costeffective fabrication method for architectural designs of complex shapes. By moving a heated flexible metal rod though a block of expanded polystyrene, it is possible to produce shapes with both positive and negative Gaussian curvature, either for direct use or for use as moulds for concrete casting. If we can control the shape of the rod, while moving, we can produce prescribed shapes.

The flexible rod assumes at all times the shape of an Euler elastica (or elastic curve). The elastica are given in closed analytic form using elliptic functions. We use a gradient-driven optimization to approximate arbitrary planar curves by planar elastic curves. The method depends on an explicit parameterization of the space of elastic curves and on a method for finding a good initial guess for the optimization.

We approximate CAD surfaces by first extracting a collection of planar surface curves and approximating these by elastica. Providing the data for these curves to robots holding the flexible rod, we can produce an elastica-foliated surface that approximates the given CAD surface. Since not all surfaces can be closely approximated by an elastica-foliated...
surface, an arbitrary CAD surface must first be subdivided into segments that can be approximated. We discuss strategies for subdividing an arbitrary surface into segments that can be closely approximated, taking into account the aesthetics of the segmentation and the production constraints. If the given surface is smooth, we want the approximating surface to be smooth as well, so we must ensure smooth transition between the surface segments of the final result.

As an alternative to rationalization of arbitrary designs, we also present a method for direct generation of design surfaces using foliated Euler elastica. Here we work from a grid of blocks, so the segmentation is given, but we must still ensure smooth transition between segments.

**General information**
State: Published
Organisations: Department of Applied Mathematics and Computer Science, Mathematics
Authors: Nørbjerg, T. B. (Intern), Gravesen, J. (Intern), Brander, D. (Intern)
Number of pages: 101
Publication date: 2017

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Original language: English

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ISSN: 0909-3192
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Electronic versions: phd419_Norbjerg_TB.pdf

**Relations**
Projects:
Rationalization in architecture with surfaces foliated by elastic curves
Publication: Research › Ph.D. thesis – Annual report year: 2017

**Approximation by planar elastic curves**
We give an algorithm for approximating a given plane curve segment by a planar elastic curve. The method depends on an analytic representation of the space of elastic curve segments, together with a geometric method for obtaining a good initial guess for the approximating curve. A gradient-driven optimization is then used to find the approximating elastic curve.

**General information**
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Organisations: Department of Applied Mathematics and Computer Science, Mathematics
Authors: Brander, D. (Intern), Gravesen, J. (Intern), Nørbjerg, T. B. (Intern)
Number of pages: 19
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Main Research Area: Technical/natural sciences

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ISSN (Print): 1019-7168
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Web of Science (2017): Indexed Yes
BFI (2016): BFI-level 1
Scopus rating (2016): SJR 0.848 SNIP 1.06 CiteScore 1.3
Web of Science (2016): Indexed yes
BFI (2015): BFI-level 1
Scopus rating (2015): SJR 1.161 SNIP 1.354 CiteScore 1.33
Web of Science (2015): Indexed yes
BFI (2014): BFI-level 1
Scopus rating (2014): SJR 1.307 SNIP 1.54 CiteScore 1.57
Web of Science (2014): Indexed yes
This paper proposes a novel pre-processing method for industrial robotic CNC-milling. The method targets a hybrid machining process, in which the main bulk of material is removed through robotic hot or abrasive wire cutting, after which regular CNC-machining is employed for removal of the remaining material volume. Hereby, the roughing process is significantly sped up, reduc-ing overall machining time. We compare our method to the convex hull and re-move between 5% and 75% more material; on most models we obtain a 50% improvement. Our method ensures that no overcutting happens and that the result is cuttable by wire cutting.

**Cuttable Ruled Surface Strips for Milling**

This paper proposes a novel pre-processing method for industrial robotic CNC-milling. The method targets a hybrid machining process, in which the main bulk of material is removed through robotic hot or abrasive wire cutting, after which regular CNC-machining is employed for removal of the remaining material volume. Hereby, the roughing process is significantly sped up, reduc-ing overall machining time. We compare our method to the convex hull and re-move between 5% and 75% more material; on most models we obtain a 50% improvement. Our method ensures that no overcutting happens and that the result is cuttable by wire cutting.

**General information**

State: Published
Organisations: Department of Applied Mathematics and Computer Science, Image Analysis & Computer Graphics, Mathematics, Odico Formwork Robotics Aps
Authors: Steenstrup, K. H. (Intern), Nørbjerg, T. B. (Intern), Søndergaard, A. (Ekstern), Bærentzen, J. A. (Intern), Gravesen, J. (Intern)
Pages: 328-342
Publication date: 2016

**Host publication information**
Designing for hot-blade cutting: Geometric Approaches for High-Speed Manufacturing of Doubly-Curved Architectural Surfaces

In this paper we present a novel method for the generation of doubly-curved, architectural design surfaces using swept Euler elastica and cubic splines. The method enables a direct design to production workflow with robotic hot-blade cutting, a novel robotic fabrication method under development by authors of the paper, which facilitates high-speed production of doubly-curved foam moulds. Complementary to design rationalisation, in which arbitrary surfaces are translated to hot-blade-cuttable geometries, the presented method enables architects and designers to design directly with the non-trivial constraints of blade-cutting in a bottom-up fashion, enabling an exploration of the unique architectural potential of this fabrication approach. The method is implemented as prototype design tools in MatLAB, C++, GhPython, and Python and demonstrated through cutting of expanded polystyrene foam design examples.

General information
State: Published
Authors: Brander, D. (Intern), Bærentzen, J. A. (Intern), Clausen, K. (Ekstern), Fisker, A. (Intern), Gravesen, J. (Intern), Lund, M. N. (Ekstern), Narbjerg, T. B. (Intern), Steenstrup, K. H. (Intern), Søndergaard, A. (Ekstern)
Pages: 306-327
Publication date: 2016

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Title of host publication: Advances in Architectural Geometry 2016
Publisher: vdf Hochschulverlag AG an der ETH Zürich
Editors: Adriaenssens, S., Gramazio, F., Kohler, M., Menges, A., Pauly, M.
ISBN (Print): 978-3-7281-3778-4
Main Research Area: Technical/natural sciences
Robotic fabrication, Hot blade, Digital design, EPS-moulds, Cost-efficiency, Concrete structures
Electronic versions:
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DOIs:
10.3218/3778-4_21
Links:
Source: PublicationPreSubmission
Source-ID: 125814394
Publication: Research - peer-review › Article in proceedings – Annual report year: 2016

Hot Blade Cuttings for the Building Industries
The constructions of advanced architectural designs are presently very labour intensive, time consuming, and expensive. They are therefore only applied to a few prestige projects, and it is a major challenge for the building industry to bring the costs down and thereby offer the architects more variability in the (economically allowed) designs - i.e., to allow them to think out of the box. To address this challenge The Danish National Advanced Technology Foundation (now InnovationsFonden) is currently supporting the BladeRunner project that involves several Danish companies and public institutions. The project aims to reduce the amount of manual labour as well as production time by applying robots to cut expanded polystyrene (EPS) moulds for the concrete to form doubly curved surfaces. The scheme is based upon the so-
called Hot Wire or Hot Blade technology where the surfaces are essentially swept out by driving an Euler elastica through a block of EPS. This paper will be centered around the mathematical challenges encountered in the implementation of this idea. Since the elastica themselves are well known and described in the works of Euler et al. already in eighteenth century, these new challenges are mainly concerned with the rationalization of the architects’ CAD drawings into surfaces that can be created via this particular sweeping and cutting technology.

**General information**

**State:** Published

**Organisations:** Department of Applied Mathematics and Computer Science, Mathematics, Image Analysis & Computer Graphics, Norwegian University of Science and Technology, Autodesk ApS

**Authors:** Brander, D. (Intern), Bærentzen, J. A. (Intern), Evgrafov, A. (Ekstern), Gravesen, J. (Intern), Markvorsen, S. (Intern), Nørberg, T. B. (Intern), Nørbjerg, T. B. (Intern), Nørtoft, P. (Intern), Steenstrup, K. H. (Intern)

**Number of pages:** 19

**Publication date:** 2016

**Host publication information**

**Title of host publication:** Proceedings of the KoMSO Challenge Workshop: Math for the Digital Factory (2014)

**Publisher:** Springer

**Editors:** Ghezzi, L., Hömberg, D., Landry, C.

**Main Research Area:** Technical/natural sciences

**Workshop:** KoMSO Challenge Workshop, Berlin, Germany, 07/05/2014 - 07/05/2014

**Publication:** Research - peer-review › Article in proceedings – Annual report year: 2016

**Robotic Hot-Blade Cutting: An Industrial Approach to Cost-Effective Production of Double Curved Concrete Structures**

This paper presents a novel method for cost-effective, robotic production of double curved formwork in Expanded Polystyrene (EPS) for in situ and prefabricated concrete construction. A rationalization and segmentation procedure is developed, which allows for the transliteration of double curved NURBS surfaces to Euler elastica surface segments, while respecting various constraints of production. An 18 axis, tri-robot system approximates double curved NURBS surfaces by means of an elastically deformed and heated blade, mounted on the flanges of two manipulators. Re-orienting or translating either end of the blade dynamically deforms the blade’s curvature. The blade follows the contours of the rationalized surface by continuous change in position and orientation of the end-effectors. The concept’s potential is studied by a pilot production of a full-scale demonstrator panel assembly.

**General information**

**State:** Published

**Organisations:** Department of Applied Mathematics and Computer Science, Mathematics, Image Analysis & Computer Graphics, Department of Mechanical Engineering, Manufacturing Engineering, Odico Formwork Robotics Aps, GXN A/S, Danish Technological Institute

**Authors:** Søndergaard, A. (Ekstern), Feringa, J. (Ekstern), Nørbjerg, T. B. (Intern), Steenstrup, K. H. (Intern), Brander, D. (Intern), Gravesen, J. (Intern), Markvorsen, S. (Intern), Bærentzen, J. A. (Intern), Petkov, K. (Intern), Hattel, J. H. (Intern), Clausen, K. (Ekstern), Jensen, K. (Ekstern), Knudsen, L. (Ekstern), Kortbek, J. (Ekstern)

**Pages:** 150-164

**Publication date:** 2016

**Host publication information**

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**Publisher:** Springer

**Editors:** Reinhardt, D., Saunders, R., Burry, J.

**ISBN (Print):** 978-3-319-26376-2

**ISBN (Electronic):** 978-3-319-26378-6

**Main Research Area:** Technical/natural sciences

**Robotic fabrication, Hot-Blade, EPS-molds, Cost-efficiency, concrete structures**

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10.1007/978-3-319-26378-6_11

**Source:** PublicationPreSubmission

**Source-ID:** 125390367

**Publication:** Research - peer-review › Book chapter – Annual report year: 2016

**Projects:**

**BladeRunner - Applied Geometry**

Department of Applied Mathematics and Computer Science
Period: 01/06/2013 → 26/10/2016
Number of participants: 6
Phd Student:
Nørbjerg, Toke Bjerge (Intern)
Supervisor:
Brander, David (Intern)
Main Supervisor:
Gravesen, Jens (Intern)
Examiner:
Røgen, Peter (Intern)
Polthier, Konrad (Ekstern)
Wallner, Johannes (Ekstern)

Financing sources
Source: Internal funding (public)
Name of research programme: 1/3 FUU, 1/3 inst 1/3 Andet

Relations
Publications:
Rationalization in architecture with surfaces foliated by elastic curves
Project: PhD

Activities:

Advances in Architectural Geometry 2014
Period: 18 Sep 2014 → 21 Sep 2014
Toke Bjerge Nørbjerg (Participant)
Department of Applied Mathematics and Computer Science
Mathematics

Description
Participation in conference and presenting poster

Advances in Architectural Geometry conference
Documents:
AAGposter
Links:
http://www.architecturalgeometry.org/aag14/ (Conference homepage)

Related event

Advances in Architectural Geometry 2014
18/09/2014 → 21/09/2014
London, United Kingdom
Activity: Attending an event › Participating in or organising a conference

Technische Universität Graz
Period: 13 Sep 2014 → 13 Feb 2015
Toke Bjerge Nørbjerg (Visiting researcher)
Department of Applied Mathematics and Computer Science
Mathematics

Description
PhD research stay at the Institute of Geometry at TU Graz
Activity: Visiting an external institution › Visiting another research institution

SIAM conference on Geometric and Physical Modeling
Period: 12 Nov 2013 → 14 Nov 2013
Toke Bjerge Nørbjerg (Participant)
Department of Applied Mathematics and Computer Science
Mathematics

**Description**
Participation in conference and contribution to poster session

SIAM conference on Geometric and Physical Modeling
Documents:
Poster_for_SIAM_conference

Links:
http://www.siam.org/meetings/gdspm13/ (Conference homepage)

**Related event**

**SIAM conference on Geometric and Physical Modeling**
11/11/2013 → 14/11/2013
Denver, Colorado, United States
Activity: Attending an event › Participating in or organising a conference