**Bifurcation theory for vortices with application to boundary layer eruption**

We develop a bifurcation theory describing the conditions under which vortices are created or destroyed in a two-dimensional incompressible flow. We define vortices using the $\omega$-criterion and analyse the vortex structure by considering the evolution of the zero contours of $\omega$. The theory identifies topological changes of the vortex structure and classifies these as four possible types of bifurcations, two occurring away from boundaries, and two occurring near no-slip walls. Our theory provides a description of all possible codimension-one bifurcations where time is treated as the bifurcation parameter. To illustrate our results, we consider the early stages of boundary layer eruption at moderate Reynolds numbers in the range from $Re$ to $Re$. By analysing numerical simulations of the phenomenon, we show how to describe the eruption process as sequences of the four possible bifurcations of codimension one. Our simulations show that there is a single codimension-two point within our parameter range. This codimension-two point arises at via the coalescence of two codimension-one bifurcations which are associated with the creation and subsequent destruction of one of the vortices that erupt from the boundary layer. We present a theoretical description of this process and explain how the occurrence of this phenomenon separates the parameter space into two regions with distinct evolution of the topology of the vortices.

**General information**

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Organisations: Department of Applied Mathematics and Computer Science, Mathematics, University of Manchester, Roskilde University  
Corresponding author: Brons, M.  
Contributors: Nielsen, A. R., Heil, M., Andersen, M., Brons, M.  
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Vorticity topology of vortex pair interactions at low Reynolds numbers
We investigate vortex merging at low Reynolds numbers from a topological point of view. We identify vortices as local extremal points of vorticity and follow the motion and bifurcation of these points as time progresses. We consider both two-dimensional simulations of the vorticity transport equation and an analytical study of the core growth model. The merging process of identical vortices is shown to occur through a pitchfork bifurcation and for asymmetric vortices one vortex merges with a saddle through a cusp (perturbed pitchfork) bifurcation. Excellent agreement between the core growth model and the numerical simulations is observed. For higher Reynolds numbers, filamentation becomes dominant hence limiting the predictive value of the core growth model. A complete investigation of merging in the core growth model is conducted for all possible vortex strengths. Simple, analytical expressions are derived for bifurcation curves, merging time, and vortex positions depending on systems parameters.

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Organisations: Department of Applied Mathematics and Computer Science, Mathematics, Roskilde University, Ecole Nationale Supérieure de Mécanique et Aérotechnique
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Organisations: Department of Applied Mathematics and Computer Science, Mathematics, Technion-Israel Institute of Technology, Sam Houston State University, Tel Aviv University
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Source-ID: 2437905951
Research output: Contribution to journal ➔ Editorial – Annual report year: 2018 ➔ Research ➔ peer-review
Canards in stiction: on solutions of a friction oscillator by regularization

We study the solutions of a friction oscillator subject to stiction. This discontinuous model is non-Filippov, and the concept of Filippov solution cannot be used. Furthermore some Carathéodory solutions are unphysical. Therefore we introduce the concept of stiction solutions: these are the Carathéodory solutions that are physically relevant, i.e. the ones that follow the stiction law. However, we find that some of the stiction solutions are forward non-unique in subregions of the slip onset. We call these solutions singular, in contrast to the regular stiction solutions that are forward unique. In order to further the understanding of the non-unique dynamics, we introduce a regularization of the model. This gives a singularly perturbed problem that captures the main features of the original discontinuous problem. We identify a repelling slow manifold that separates the forward slipping to forward sticking solutions, leading to a high sensitivity to the initial conditions. On this slow manifold we find canard trajectories, that have the physical interpretation of delaying the slip onset. We show with numerics that the regularized problem has a family of periodic orbits interacting with the canards. We observe that this family has a saddle stability and that it connects, in the rigid body limit, the two regular, slip-stick branches of the discontinuous problem, that were otherwise disconnected.

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Contributors: Bossolini, E., Brøns, M., Kristiansen, K. U.
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Web of Science (2017): Indexed yes
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On the approximation of the canard explosion point in singularly perturbed systems without an explicit small parameter

A canard explosion is the dramatic change of period and amplitude of a limit cycle of a system of nonlinear ODEs in a very narrow interval of the bifurcation parameter. It occurs in slow–fast systems and is well understood in singular perturbation problems where a small parameter epsilon defines the time-scale separation. We present an iterative algorithm for the determination of the canard explosion point which can be applied for a general slow–fast system without an explicit small parameter. We also present assumptions under which the algorithm gives accurate estimates of the canard explosion point. Finally, we apply the algorithm to the van der Pol equations, a Templator model for a self-replicating system and a model for intracellular calcium oscillations with no explicit small parameters and obtain very good agreement with results from numerical simulations.

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Contributors: Brøns, M., Kristiansen, K. U.
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Singular limit analysis of a model for earthquake faulting
In this paper we consider the one dimensional spring-block model describing earthquake faulting. By using geometric singular perturbation theory and the blow-up method we provide a detailed description of the periodicity of the earthquake episodes. In particular, the limit cycles arise from a degenerate Hopf bifurcation whose degeneracy is due to an underlying Hamiltonian structure that leads to large amplitude oscillations. We use a Poincaré compactification to study the system near infinity. At infinity the critical manifold loses hyperbolicity with an exponential rate. We use an adaptation of the blow-up method to recover the hyperbolicity. This enables the identification of a new attracting manifold that organises the dynamics at infinity. This in turn leads to the formulation of a conjecture on the behaviour of the limit cycles as the time-scale separation increases. We provide the basic foundation for the proof of this conjecture and illustrate our findings with numerics.

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Sparse identification of a predator-prey system from simulation data of a convection model
The use of low-dimensional dynamical systems as reduced models for plasma dynamics is useful as solving an initial value problem requires much less computational resources than fluid simulations. We utilize a data-driven modeling approach to identify a reduced model from simulation data of a convection problem. A convection model with a pressure source centered at the inner boundary models the edge dynamics of a magnetically confined plasma. The convection problem undergoes a sequence of bifurcations as the strength of the pressure source increases. The time evolution of the
energies of the pressure profile, the turbulent flow, and the zonal flow capture the fundamental dynamic behavior of the full system. By applying the sparse identification of nonlinear dynamics (SINDy) method, we identify a predator-prey type dynamical system that approximates the underlying dynamics of the three energy state variables. A bifurcation analysis of the system reveals consistency between the bifurcation structures, observed for the simulation data, and the identified underlying system.

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**Topological bifurcations in the evolution of coherent structures in a convection model**
Blob filaments are coherent structures in a turbulent plasma flow. Understanding the evolution of these structures is important to improve magnetic plasma confinement. Three state variables describe blob filaments in a plasma convection model. A dynamical systems approach analyzes the evolution of these three variables. A critical point of a variable defines a feature point for a region where that variable is significant. For a range of Rayleigh and Prandtl numbers, the bifurcations of the critical points of the three variables are investigated with time as the primary bifurcation parameter. Bifurcation curves separate the parameter planes into regions with different critical point configurations for the state variables. For Prandtl number equal to 1, the number of critical points of each state variable increases with increasing Rayleigh number. For Rayleigh number equal to 104, the number of critical points is the greatest for Prandtl numbers of magnitude 100.

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Topological fluid mechanics of the formation of the Kármán-vortex street
We explore the two-dimensional flow around a circular cylinder with the aim of elucidating the changes in the topology of the vorticity field that lead to the formation of the Kármán vortex street. Specifically, we analyse the formation and disappearance of extremal points of vorticity, which we consider to be feature points for vortices. The basic vortex creation mechanism is shown to be a topological cusp bifurcation in the vorticity field, where a saddle and an extremum of the vorticity are created simultaneously. We demonstrate that vortices are first created approximately 100 diameters downstream of the cylinder, at a Reynolds number, ReK, which is slightly larger than the critical Reynolds number, Recrit≈46, at which the flow becomes time periodic. For Re slightly above ReK, the newly created vortices disappear again a short distance further downstream. As Re is further increased, the points of creation and disappearance move rapidly upstream and downstream, respectively, and the Kármán vortex street persists over increasingly large streamwise distances.

Bifurcations of a creeping air–water flow in a conical container
This numerical study describes the eddy emergence and transformations in a slow steady axisymmetric air–water flow, driven by a rotating top disk in a vertical conical container. As water height (Formula presented.) and cone half-angle (Formula presented.) vary, numerous flow metamorphoses occur. They are investigated for (Formula presented.), and (Formula presented.). For small (Formula presented.), the air flow is multi-cellular with clockwise meridional circulation near the disk. The air flow becomes one cellular as (Formula presented.) exceeds a threshold depending on (Formula presented.). For all (Formula presented.), the water flow has an unbounded number of eddies whose size and strength diminish as the cone apex is approached. As the water level becomes close to the disk, the outmost water eddy with clockwise meridional circulation expands, reaches the interface, and induces a thin layer with anticlockwise circulation in the air. Then this layer expands and occupies the entire air domain. The physical reasons for the flow transformations are provided. The results are of fundamental interest and can be relevant for aerial bioreactors.
Patterns of a slow air-water flow in a semispherical container

This numerical study analyzes the development of eddies in a slow steady axisymmetric air-water flow in a sealed semispherical container, driven by a rotating top disk. As the water height, $H_w$, increases, new flow cells emerge in both water and air. First, an eddy emerges near the axis-bottom intersection. Then this eddy expands and reaches the interface, inducing a new cell in the air flow. This cell appears as a thin near-axis layer which then expands and occupies the entire air domain. As the disk rotation intensifies at $H_w = 0.8$, the new air cell shrinks to the axis and disappears. The bulk water circulation becomes separated from the interface by a thin layer of water counter-circulation. These changes in the flow topology occur due to (a) competing effects of the air meridional flow and swirl, which drive meridional motions of opposite directions in water, and (b) feedback of water flow on the air flow. In contrast to flows in cylindrical and conical containers, there is no interaction with Moffatt corner vortices here.
Sixth International Symposium on Bifurcations and Instabilities in Fluid Dynamics (BIFD2015): Foreword

Hydrodynamic stability is of fundamental importance in fluid dynamics. As a well-established subject of scientific investigation, it continues to attract great interest in the fluid mechanics community. Bifurcations and instabilities are observed in all areas of fundamental and applied fluid dynamics and remain a challenge for experimental, theoretical and computational studies. Examples of prototypical hydrodynamic instabilities are the Rayleigh–Bénard, Taylor–Couette, Bénard–Marangoni, Rayleigh–Taylor, and Kelvin–Helmholtz instabilities. A fundamental understanding of bifurcation patterns requires the identification of mechanisms responsible for the instability. From an applied point of view, such knowledge is also necessary in order to design reliable and efficient industrial processes, such as melting, mixing, crystal growth, coating, and welding. Modeling of instability mechanisms in biological and biomedical devices is currently a very active and rapidly developing area of research with important biotechnological and medical applications, such as biofilm engineering and wound healing. The understanding of symmetry-breaking in hemodynamics could have important consequences for vascular diseases, such as atherosclerotic and vulnerable plaques, abdominal aortic aneurisms, carotid artery disease, and pulmonary embolisms and implications for vascular interventions such as grafting and stenting. The collection of papers in this issue is a selection of the presentations given at the Sixth International Symposium on Instability and Bifurcations in Fluid Dynamics (BIFD) held at the ESPCI, Paris, 15–17 July 2015. With four invited and nearly 400 contributed talks, the symposium gave an overview of the state of the art of the field including experimental, theoretical, and computational approaches to convection, effects of magnetic fields, wake flows, rotating flows, and many other problems. The complete program can be found at the conference website http://bifd2015.sciencesconf.org/.

Bifurcation Analysis of Structures in a Convection Model

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Bifurcation Analysis of Structures in a Convection Model

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Organisations: Department of Applied Mathematics and Computer Science, Mathematics, Technion-Israel Institute of Technology, School of Mechanical Engineering, ESPCI, Department of Physics
Contributors: Dam, M., Brøns, M., Rasmussen, J. J., Naulin, V.
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Publication date: 2015
Peer-reviewed: Yes
Codimension three bifurcation of streamline patterns close to a no-slip wall: A topological description of boundary layer eruption

A vortex close to a no-slip wall gives rise to the creation of new vorticity at the wall. This vorticity may organize itself into vortices that erupt from the separated boundary layer. We study how the eruption process in terms of the streamline topology is initiated and varies in dependence of the Reynolds number Re. We show that vortex structures are created in the boundary layer for Re around 600, but that these disappear again without eruption unless Re > 1000. The eruption process is topologically unaltered for Re up to 5000. Using bifurcation theory, we obtain a topological phase space for the eruption process, which can account for all observed changes in the Reynolds number range we consider. The bifurcation diagram complements previously analyzes such that the classification of topological bifurcations of flows close to no-slip walls with up to three parameters is now complete.

Extending the zero-derivative principle for slow–fast dynamical systems

Slow–fast systems often possess slow manifolds, that is invariant or locally invariant sub-manifolds on which the dynamics evolves on the slow time scale. For systems with explicit timescale separation, the existence of slow manifolds is due to Fenichel theory, and asymptotic expansions of such manifolds are easily obtained. In this paper, we discuss methods of approximating slow manifolds using the so-called zero-derivative principle. We demonstrate several test functions that work for systems with explicit time scale separation including ones that can be generalized to systems without explicit timescale separation. We also discuss the possible spurious solutions, known as ghosts, as well as treat the Templator system as an example.
Mixed-Mode Oscillations Due to a Singular Hopf Bifurcation in a Forest Pest Model

In a forest pest model, young trees are distinguished from old trees. The pest feeds on old trees. The pest grows on a fast scale, the young trees on an intermediate scale, and the old trees on a slow scale. A combination of a singular Hopf bifurcation and a "weak return" mechanism, characterized by a small change in one of the variables, determines the features of the mixed-mode oscillations. Period-doubling and saddle-node bifurcations lead to closed families (called isolas) of periodic solutions in a bifurcation corresponding to a singular Hopf bifurcation.

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Web of Science (2015): Indexed yes
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Predicting optimal back-shock times in ultrafiltration hollow fiber modules II: Effect of inlet flow and concentration dependent viscosity

This paper concerns mathematical modeling and computational fluid dynamics of back-shocking during hollow fibre ultrafiltration of dextran T500. In this paper we present a mathematical model based on first Principles, i.e., solving the Navier-Stokes equation along with the continuity equation for both the solute and the solvent. We investigate the validity of the estimate on the optimal back-shock time, i.e., the back-shock time needed to achieve the highest permeate flux published in a previous paper by the authors (Vinther et al., Predicting optimal back-shock times in ultrafiltration hollow fibre membranes, J. Membr. Sci. 470 (2014) 275-293 [33]). Furthermore, the simulations have been performed with two different inlet velocities, i.e., crossflow velocities and are clone with and without a concentration dependent viscosity. This enables us, for the first time, to investigate the effect of different inlet velocities and the effect of a concentration polarization on the observed rejection and the permeate flux, as a function of different back-shock times. In all cases the average permeate flux and the observed rejection during one period of back-shocking were found to be higher than the steady-state values - representing the long time behavior of a similar separation process performed without back-shocking - when using the optimal back-shock time. It is concluded that the estimate of the optimal back-shock time is in good agreement with the optimal time found in the simulations performed in this paper. Furthermore, it is found that the optimal back-shock time increases when the viscosity is allowed to depend on the concentration. It is found that this can be explained by a decrease in the velocity tangential to the membrane due to the increase in viscosity where the concentration is high - resulting in a longer time for the concentration polarization to be convected tangentially along the membrane surface. The ratio between the average flux over a back-shock cycle and the steady-state flux is found to increase with increasing inlet velocity. Furthermore, this ratio increases when the viscosity depends on the concentration. This is due to the relatively lower steady-state value when the viscosity depends on the concentration. Moreover, an increase in observed rejection is found when using back-shocking. The increase in observed rejection is found to be largest when the inlet velocity is high and the viscosity depends on the concentration. (C) 2015 Elsevier B.V. All rights reserved.

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Contributors: Vinther, F., Pinelo, M., Brøns, M., Jonsson, G. E., Meyer, A. S.
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Vortex breakdown in a truncated conical bioreactor
This numerical study explains the eddy formation and disappearance in a slow steady axisymmetric air–water flow in a vertical truncated conical container, driven by the rotating top disk. Numerous topological metamorphoses occur as the water height, Hw, and the bottom-sidewall angle, α, vary. It is found that the sidewall convergence (divergence) from the top to the bottom stimulates (suppresses) the development of vortex breakdown (VB) in both water and air. At α = 60°, the flow topology changes eighteen times as Hw varies. The changes are due to (a) competing effects of AMF (the air meridional flow) and swirl, which drive meridional motions of opposite directions in water, and (b) feedback of water flow on AMF. For small Hw, the AMF effect dominates. As Hw increases, the swirl effect dominates and causes VB. The water flow feedback produces and modifies air eddies. The results are of fundamental interest and can be relevant for aerial bioreactors.

General information
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Research output: Contribution to journal › Journal article – Annual report year: 2014 › Research › peer-review
An Iterative Method for the Approximation of Fibers in Slow-Fast Systems

In this paper we extend a method for iteratively improving slow manifolds so that it also can be used to approximate the fiber directions. The extended method is applied to general finite-dimensional real analytic systems where we obtain exponential estimates of the tangent spaces to the fibers. The method is demonstrated on the Michaelis–Menten–Henri model and the Lindemann mechanism. The latter example also serves to demonstrate the method on a slow-fast system in nonstandard slow-fast form. Finally, we extend the method further so that it also approximates the curvature of the fibers.

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Bifurcation Analysis and Dimension Reduction of a Predator-Prey Model for the L-H Transition

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Mathematical modelling of dextran filtration through hollow fibre membranes
In this paper we present a mathematical model of an ultrafiltration process. The results of the model are produced using standard numerical techniques with Comsol Multiphysics. The model describes the fluid flow and separation in hollow fibre membranes. The flow of solute and solvent within the hollow fibre is modelled by solving the Navier-Stokes equation along with the continuity equation for both the solute and the solvent. The flux of solute and solvent through the membrane are given by the solution diffusion model, since ultrafiltration occurs at high rejections. For a given set of parameters describing the characteristics of the membrane, effect on the observed and the intrinsic rejection of the membrane are investigated for the different working parameters: inlet velocity, molecular weight, and transmembrane pressure. Furthermore, the model investigates the effect of a concentration dependent viscosity. The model shows that both the observed and intrinsic rejection increase when the inlet velocity increases. Moreover, the intrinsic rejection increases as a function of transmembrane pressure, but the observed rejection has a characteristic maximum. Therefore, the observed rejection can either increase or decrease as a function of pressure. The influence of a concentration dependent viscosity is
to increase the concentration on the membrane surface. This leads to a decrease in both the observed and the intrinsic rejection, when compared to a constant viscosity. For small values of the solute permeability the concentration dependent viscosity decreases the volumetric flux through the membrane at high pressures. This effect is due to a very high concentration at the membrane surface. The model is related to experimental data. There is a good qualitative and a reasonable quantitative agreement between simulations and experimental data.

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  Web of Science (2014): Impact factor 3.091
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Source-ID: n:oai:DTIC-ART:compendex/437073387::38202
Research output: Contribution to journal › Journal article – Annual report year: 2014 › Research › peer-review

Predicting optimal back-shock times in ultrafiltration hollow fibre modules through path-lines
This paper presents a two dimensional mathematical model of back-shocking in ultrafiltration. The model investigates the effect of back-shocking on concentration polarization. The model shows a positive effect on both the volumetric flux and the observed rejection when back-shocking is applied as compared to the steady-state solution. Furthermore, the effect of changing different parameters such as inlet velocity, forward and backwards pressure on the back-shock time, the increase in volumetric flux and observed rejection, is presented. Moreover, two analytical estimates for the optimal back-shock time derived from calculating the path-lines during a back-shock cycle are presented. Both of these expressions are in good agreement with the results obtained from the mathematical model and data collected from the literature. Based on this, a simple expression for an optimal back-shock time in a multi-parameter problem is provided.

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DOIs: 10.1016/j.memsci.2014.07.031
Rotation reversal in a 1D turbulence spreading model

General information
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Organisations: Department of Physics, Plasma Physics and Fusion Energy, Department of Applied Mathematics and Computer Science, Mathematics
Contributors: Naulin, V., Juul Rasmussen, J., Dam, M., Brøns, M.
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Research output: Chapter in Book/Report/Conference proceeding › Conference abstract in proceedings – Annual report year: 2014 › Research › peer-review

Topology of helical fluid flow
Considering a coordinate-free formulation of helical symmetry rather than more traditional definitions based on coordinates, we discuss basic properties of helical vector fields and compare results from the literature obtained with other approaches. In particular, we discuss the role of the stream function for the topology of the streamline pattern in incompressible flows. On this basis, we perform a comprehensive study of the topology of the flow field generated by a helical vortex filament in an ideal fluid. The classical expression for the stream function obtained by Hardin (Hardin, J. C. 1982 Phys. Fluids 25, 1949-1952) contains an infinite sum of modified Bessel functions. Using the approach by Okulov (Okulov, V. L. 1995 Russ. J. Eng. Thermophys. 5, 63-75) we obtain a closed-form approximation which is considerably easier to analyse. Critical points of the stream function can be found from the zeroes of a single real function of one variable, and we show that three different flow topologies can occur, depending on a single dimensionless parameter. By including the self-induced velocity on the vortex filament by a localised induction approximation, the stream function is slightly modified and an extra parameter is introduced. In this setting two new flow topologies arise, but not more than two critical points occur for any combination of parameters.

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Publication status: Published
Organisations: Department of Applied Mathematics and Computer Science, Mathematics
Contributors: Andersen, M., Brøns, M.
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Scopus rating (2014): CiteScore 0.93 SJR 0.765 SNIP 0.9
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Web of Science (2014): Indexed yes
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Keywords: Helical flow, Flow topology, Vortex filaments, Asymptotic expansions
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Research output: Contribution to journal › Journal article – Annual report year: 2014 › Research › peer-review
Vorticity generation and conservation for two-dimensional interfaces and boundaries
The generation, redistribution and, importantly, conservation of vorticity and circulation is studied for incompressible Newtonian fluids in planar and axisymmetric geometries. A generalised formulation of the vorticity at the interface between two fluids for both no-slip and stress-free conditions is presented. Illustrative examples are provided for planar Couette flow, Poiseuille flow, the spin-up of a circular cylinder, and a cylinder below a free surface. For the last example, it is shown that, although large imbalances between positive and negative vorticity appear in the wake, the balance is found in the vortex sheet representing the stress-free surface.

General information
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Organisations: Department of Applied Mathematics and Computer Science, Mathematics, Monash University, Aix Marseille Université
Contributors: Brøns, M., Thompson, M. C., Leweke, T., Hourigan, K.
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An iterative method for the canard explosion in general planar systems
The canard explosion is the change of amplitude and period of a limit cycle born in a Hopf bifurcation in a very narrow parameter interval. The phenomenon is well understood in singular perturbation problems where a small parameter controls the slow/fast dynamics. However, canard explosions are also observed in systems where no such parameter can obviously be identified. Here we show how the iterative method of Roussel and Fraser, devised to construct regular slow manifolds, can be used to determine a canard point in a general planar system of nonlinear ODEs. We demonstrate the method on the van der Pol equation, showing that the asymptotics of the method is correct, and on a templator model for a self-replicating system.

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Contributors: Brøns, M.
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Research output: Chapter in Book/Report/Conference proceeding › Article in proceedings – Annual report year: 2013 › Research › peer-review
Bifurcation analysis and dimension reduction of a predator-prey model for the L-H transition

The L-H transition denotes a shift to an improved confinement state of a toroidal plasma in a fusion reactor. A model of the L-H transition is required to simulate the time dependence of tokamak discharges that include the L-H transition. A 3-ODE predator-prey type model of the L-H transition is investigated with bifurcation theory of dynamical systems. The analysis shows that the model contains three types of transitions: an oscillating transition, a sharp transition with hysteresis, and a smooth transition. The model is recognized as a slow-fast system. A reduced 2-ODE model consisting of the full model restricted to the flow on the critical manifold is found to contain all the same dynamics as the full model. This means that all the dynamics in the system is essentially 2-dimensional, and a minimal model of the L-H transition could be a 2-ODE model.

Recent progress in the relative equilibria of point vortices — In memoriam Hassan Aref

Hassan Aref, who sadly passed away in 2011, was one of the world's leading researchers in the dynamics and equilibria of point vortices. We review two problems on the subject of point vortex relative equilibria in which he was engaged at the time of his death: bilinear relative equilibria and the geometry of the three-vortex problem as it relates to equilibria. A set of point vortices is in relative equilibrium if it is at most rotating rigidly around the center of vorticity, and the configuration is bilinear if the vortices are placed on two orthogonal lines in the co-rotating frame. A very complete characterisation of the bilinear case can be obtained when one of the lines contains only two vortices. The classic three-vortex problem can be viewed anew by considering the dynamics of the circle circumscribing the vortex triangle and the interior angles of that triangle. This approach leads naturally to the observation that the equilateral triangle is the only equilibrium configuration for three point vortices, regardless of their strength values.
Vorticity generation and wake transition for a translating circular cylinder: Wall proximity and rotation effects

The wake transitions of generic bluff bodies, such as a circular cylinder, near a wall are important because they provide understanding of different transition paths towards turbulence, and give some insight into the effect of surface modifications on the flow past larger downstream structures. In this article, the fundamentals of vorticity generation and transport for the two-dimensional flow of incompressible Newtonian fluids are initially reviewed. Vorticity is generated only at boundaries by tangential pressure gradients or relative acceleration. After generation, it can cross-annihilate with opposite-signed vorticity, and can be stored at a free surface, thus conserving the total vorticity, or circulation. Vorticity generation, diffusion and storage are demonstrated for a cylinder translating and rotating near a wall. The wake characteristics and the wake transitions are shown to change dramatically under the influence of cylinder rotation and wall proximity. At gaps between the cylinder and the wall of less than approximately 0.25 cylinder diameter, the wake becomes three dimensional prior to becoming unsteady, while for larger gaps the initial transition is to an unsteady two-dimensional wake. At a gap of 0.3 cylinder diameter, we observe a sharp increase in the critical Reynolds number at which three-dimensionality sets in. As the gap is further increased, the critical Reynolds number initially decreases before increasing to that for an isolated cylinder. The effect of cylinder rotation on these transitions is also quantified, with forward (prograde) rotation enhancing three-dimensional instability and reverse (retrograde) rotation stabilising the wake. High retrograde rotation leads to suppression of three-dimensional flow until beyond the highest Reynolds number investigated (Re=750).

General information
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Organisations: Department of Applied Mathematics and Computer Science, Mathematics, Monash University, Aix Marseille Universite
Contributors: Hourigan, K., Rao, A., Brøns, M., Leweke, T., Thompson, M., Yaojun Ge, S. C.
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Source-ID: n:oai:DTIC-ART:elsevier/409663269::33112
Research output: Contribution to journal › Journal article – Annual report year: 2013 › Research › peer-review

An iterative method for the canard explosion in general planar systems
The canard explosion is the change of amplitude and period of a limit cycle born in a Hopf bifurcation in a very narrow parameter interval. The phenomenon is well understood in singular perturbation problems where a small parameter controls the slow/fast dynamics. However, canard explosions are also observed in systems where no such parameter is present. Here we show how the iterative method of Roussel and Fraser, devised to construct regular slow manifolds, can be used to determine a canard point in a general planar system of nonlinear ODEs. We demonstrate the method on the van der Pol equation, showing that the asymptotics of the method is correct, and on a templator model for a self-
Bilinear Relative Equilibria of Identical Point Vortices

A new class of bilinear relative equilibria of identical point vortices in which the vortices are constrained to be on two perpendicular lines, conveniently taken to be the x- and y-axes of a Cartesian coordinate system, is introduced and studied. In the general problem we have m vortices on the y-axis and n on the x-axis. We define generating polynomials q(z) and p(z), respectively, for each set of vortices. A second-order, linear ODE for p(z) given q(z) is derived. Several results relating the general solution of the ODE to relative equilibrium configurations are established. Our strongest result, obtained using Sturm’s comparison theorem, is that if p(z) satisfies the ODE for a given q(z) with its imaginary zeros symmetric relative to the x-axis, then it must have at least n−m+2 simple, real zeros. For m=2 this provides a complete characterization of all zeros, and we study this case in some detail. In particular, we show that, given $q(z)=z^2+\eta^2$, where η is real, there is a unique p(z) of degree n, and a unique value of $\eta^2=A_n$, such that the zeros of q(z) and p(z) form a relative equilibrium of n+2 point vortices. We show that $A_n \approx \frac{2}{3}n + \frac{1}{2}$, as $n \to \infty$, where the coefficient of n is determined analytically, the next-order term numerically. The paper includes extensive numerical documentation on this family of relative equilibria.
Statistical modelling of the interplay between solute shape and rejection in porous membranes

The structural conformation of complex molecules, e.g., polymers and proteins, is determined by several factors like composition of the basic structural units, charge, and properties of the surrounding solvent. In absence of any chemical or physical interaction solute–solute and/or solute–membrane, it can be expected that the possibility for a solute particle to enter the membrane pore will only depend upon the relation between such molecular conformation and pore size. The objective of the present study is to use geometric and statistical modelling to determine the effect of particle elongation – from spherical to being increasingly prolate ellipsoidal – on the possibility of entering the pore, and, in turn, on the macroscopic distribution coefficient, K, and overall retention during filtration. The model showed that the value of K was maximal when the longer of the radii in the prolate ellipsoid was approximately equal to the radius of the pores, in case the spherical size of the particle was smaller than the membrane pore. Furthermore, for spherical particles larger than the pore, such a maximum was found to occur after the smaller of the radii was smaller than the pore radius. Either for spherical particles bigger or smaller than the pore radius, K was monotonically decreasing towards zero as the particles became more elongated. When relating the values of K to the friction model, the maximal rejection coefficient was found to reach a characteristic minimum when changing shape. The results suggested that the retention during porous membrane filtration can be manipulated when working with solute particles prone to alter conformation via, e.g., adding proper functional groups to the molecule, or modifying charge density/distribution by varying pH.

Canard explosion of limit cycles in templator models of self-replication mechanisms

Templators are differential equation models for self-replicating chemical systems. Beutel and Peacock-López [J. Chem. Phys. 126, 125104 (2007)] have numerically analyzed a model for a cross-catalytic self-replicating system and found two cases of canard explosion, that is, a substantial change of amplitude of a limit cycle over a very short parameter interval. We show how the model can be reduced to a two-dimensional system and how canard theory for slow-fast equations can be applied to yield analytic information about the canard explosion. In particular, simple expressions for the parameter value where the canard explosion occurs are obtained. The connection to mixed-mode oscillations also observed in the model is briefly discussed. © 2011 American Institute of Physics.
Stabilization of vortices in the wake of a circular cylinder using harmonic forcing

We explore whether vortex flows in the wake of a fixed circular cylinder can be stabilized using harmonic forcing. We use Föppl's point vortex model augmented with a harmonic point source-sink mechanism which preserves conservation of mass and leaves the system Hamiltonian. We discover a region of Lyapunov-stable vortex motion for an appropriate selection of parameters. We identify four unique parameters that affect the stability of the vortices: the uniform flow velocity, vortex equilibrium positions, forcing amplitude, and forcing frequency. We assess the robustness of the controller using a Poincaré section.

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Organisations: Department of Mathematics, Dynamical systems
Contributors: Chamoun, G. C., Schilder, F., Brøns, M.
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Publication date: 2011
Peer-reviewed: Yes

Bifurcation analysis in a vortex flow generated by an oscillatory magnetic obstacle

A numerical simulation and a theoretical model of the two-dimensional flow produced by the harmonic oscillation of a localized magnetic field (magnetic obstacle) in a quiescent viscous, electrically conducting fluid are presented. Nonuniform Lorentz forces produced by induced currents interacting with the oscillating magnetic field create periodic laminar flow patterns that can be characterized by three parameters: the oscillation Reynolds number, Re-omega, the Hartmann number, Ha, and the dimensionless amplitude of the magnetic obstacle oscillation, D. The analysis is restricted to oscillations of small amplitude and Ha=100. The resulting flow patterns are described and interpreted in terms of position and evolution of the critical points of the instantaneous streamlines. It is found that in most of the cycle, the flow is
dominated by a pair of counter rotating vortices that switch their direction of rotation twice per cycle. The transformation of the flow field present in the first part of the cycle into the pattern displayed in the second half occurs via the generation of hyperbolic and elliptic critical points. The numerical solution of the flow indicates that for low frequencies (v.e. Re-omega = 1), two elliptic and two hyperbolic points are generated, while for high frequencies (v.e. Re-omega = 100), a more complex topology involving four elliptic and two hyperbolic points appear. The bifurcation map for critical points of the instantaneous streamline is obtained numerically and a theoretical model based on a local analysis that predicts most of the qualitative properties calculated numerically is proposed.

Canards and mixed-mode oscillations in a forest pest model
We consider a three-variable forest pest model, proposed by Rinaldi & Muratori (1992) [Rinaldi, S., Muratori, S., 1992. Limit cycles in slow-fast forest-pest models. Theor. Popul. Biol. 41,26-43]. The model allows relaxation oscillations where long pest-free periods are interspersed with outbreaks of high pest concentration. For small values of the timescale of the young trees, the model can be reduced to a two-dimensional model. By a geometrical analysis we identify a canard explosion in the reduced model, that is, a change over a narrow parameter interval from outbreak dynamics to small oscillations around an endemic state. For larger values of the timescale of the young trees the two-dimensional approximation breaks down, and a broader parameter interval with mixed-mode oscillations appear, replacing the simple canard explosion. The analysis only relies on simple and generic properties of the model, and is expected to be applicable in a larger class of multiple timescale dynamical models.
Continuation of Sets of Constrained Orbit Segments

Sets of constrained orbit segments of time continuous flows are collections of trajectories that represent a whole or parts of an invariant set. A non-trivial but simple example is a homoclinic orbit. A typical representation of this set consists of an equilibrium point of the flow and a trajectory that starts close and returns close to this fixed point within finite time. More complicated examples are hybrid periodic orbits of piecewise smooth systems or quasi-periodic invariant tori. Even though it is possible to define generalised two-point boundary value problems for computing sets of constrained orbit segments, this is very disadvantageous in practice. In this talk we will present an algorithm that allows the efficient continuation of sets of constrained orbit segments together with the solution of the full variational problem.

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Organisations: Dynamical systems, Department of Mathematics, University of Illinois at Urbana-Champaign
Contributors: Schilder, F., Brøns, M., Chamoun, G. C., Dankowicz, H.
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Source: orbit
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Research output: Non-textual form → Sound/Visual production (digital) – Annual report year: 2010 → Research

Topology of vortex creation in the cylinder wake

We analyze the topology of the two-dimensional flow around a circular cylinder at moderate Reynolds numbers in the regime where the vortex wake is created. A normal form for the stream function close to the cylinder is presented and used to predict the streamline pattern both in the steady and the periodic regime, where two different vortex shedding scenarios are identified. The theoretical predictions are verified numerically. For the vorticity, a very different topology occurs with infinite nested sequences of iso-curves moving downstream. General equations of motion for critical points are derived.

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Contributors: Brøns, M., Bisgaard, A. V.
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Scopus rating (2010): SJR 1.058 SNIP 1.318
Web of Science (2010): Impact factor 1.127
Web of Science (2010): Indexed yes
Original language: English
Keywords: Vortex dynamics, Streamline patterns, Wakes, Topology, Bifurcation
Dye visualization near a three-dimensional stagnation point: application to the vortex breakdown bubble

An analytical model, based on the Fokker-Planck equation, is constructed of the dye visualization expected near a three-dimensional stagnation point in a swirling fluid flow. The model is found to predict dye traces that oscillate in density and position in the meridional plane in which swirling flows are typically visualized. Predictions based on the model are made for the steady vortex breakdown bubble in a torsionally driven cylinder and compared with computational fluid dynamics predictions and experimental observations. Previous experimental observations using tracer visualization techniques have suggested that even for low-Reynolds-number flows, the steady vortex breakdown bubble in a torsionally driven cylinder is not axisymmetric and has an inflow/outflow asymmetry at its tail. Recent numerical and theoretical studies show that the asymmetry of the vortex breakdown bubble, and consequently its open nature, can be explained by the very small imperfections that are present in any experimental rig. DISTINCT from this, here it is shown that even for a perfectly axisymmetric flow and breakdown bubble, the combined effect of dye diffusion and the inevitable small errors in the dye injection position lead to the false perception or an open bubble structure with folds near the lower stagnation point. Furthermore, the asymmetries in the predicted flow structures can be remarkably similar to those observed in flow observations and computational predictions with geometric asymmetries of the rig. Thus, when interpreting dye-visualization patterns in steady flow, even if axisymmetric flow can be achieved, it is important to take into account the relative diffusivity of the dye and the accuracy of its injection.

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Contributors: Brøns, M., Thompson, M. C., Hourigan, K.
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Scopus rating (2009): SJR 2.73 SNIP 1.936
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Original language: English
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10.1007/s00162-009-0110-0

Experimental vortex breakdown topology in a cylinder with a free surface

The free SLII-face, flow in it circular cylinder driven by a rotating bottom disk IS Studied experimentally using particle image velocimetry. Results are compared With computational,11 results assuming I stress-free surface A dye visualization Study by Spohn et al ["Observations of vortex breakdown in in open cylindrical container with I rotating bottom," Exp. Fluids 14. 70 (1993)] as well as several numerical computations. has found a range of different vortex breakdown Structures in this flow. We confirm the existence of a transition where the top of the breakdown bubble crosses from the axis to the surface, which has previously only been found numerically. We employ a technique by Brons et al ["Topology of vortex breakdown bubbles in a cylinder with rotating bottom and free surface J. Fluid Mech 428. 133 (2001)] to find the corresponding bifurcation curve in the parameter plane, which has hitherto only been used on numerical data The bifurcation Curve located here agrees well with previous numerical simulations. For low values of the Reynolds number We find a regime with vortex breakdown that has not been previously identified. Experiments deviate substantially from computations, indicating the importance of surface effects ill this regime. (C) 2009 American Institute of Physics. [doi 10.1063/1.3265718]

General information
Publication status: Published
Organisations: Dynamical systems, Department of Mathematics, Center for Fluid Dynamics, Monash University, Universite de Toulouse
A coaxial vortex ring model for vortex breakdown

Introduction to focus issue: Mixed mode oscillations: Experiment, computation, and analysis

Mixed mode oscillations (MMOs) occur when a dynamical system switches between fast and slow motion and small and large amplitude. MMOs appear in a variety of systems in nature, and may be simple or complex. This focus issue presents a series of articles on theoretical, numerical, and experimental aspects of MMOs. The applications cover physical, chemical, and biological systems. © 2008 American Institute of Physics.
Two-vortex model for vortex breakdown

A number of instability and bifurcation problems related to the dynamics of vortex wake flows are addressed using various analytical tools and approaches. We discuss the bifurcations of the streamline pattern behind a bluff body as a vortex wake is produced, a theory of the universal Strouhal-Reynolds number relation for vortex wakes, the bifurcation diagram for "exotic" wake patterns behind an oscillating cylinder first determined experimentally by Williamson & Roshko, and the bifurcations in topology of the streamlines pattern in point vortex streets. The Hamiltonian dynamics of point vortices in a periodic strip is considered. The classical results of von Kármán concerning the structure of the vortex street follow from the two-vortices-in-a-strip problem, while the stability results follow largely from a four-vortices-in-a-strip analysis. The three-vortices-in-a-strip problem is argued to be relevant to the wake behind an oscillating body.
Hopf Bifurcation, Trends, Cycles, Square Waves, and Chaos in Economic Growth Models with Time Delays

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Point Vortex Models of Bluff Body Wakes

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General information
Publication status: Published
Organisations: Dynamical systems, Department of Mathematics, Fluid Mechanics, Department of Mechanical Engineering, Center for Fluid Dynamics
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Original language: English
(Journal of Physics: Conference Series (Online); No. 64).
DOI: 10.1088/1742-6596/64/1/011001
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Streamline topology in the near wake of a circular cylinder at moderate Reynolds numbers

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Organisations: Dynamical systems, Department of Mathematics, Center for Fluid Dynamics, Roskilde University, ANSYS Europe Ltd.
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Source: orbit
Source-ID: 206491
Research output: Contribution to journal › Journal article – Annual report year: 2007 › Research › peer-review

Streamline topology: Patterns in fluid flows and their bifurcations

Using dynamical systems theory, we consider structures such as vortices and separation in the streamline patterns of fluid flows. Bifurcation of patterns under variation of external parameters is studied using simplifying normal form transformations. Flows away from boundaries, flows close to fixed walls, and axisymmetric flows are analyzed in detail. We show how to apply the ideas from the theory to analyze numerical simulations of the vortex breakdown in a closed cylindrical container.

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Contributors: Brøns, M.
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stable to an unstable slow manifold. An important ingredient of this mechanism are singularities known as folded nodes. The main focus of this article is to show how the local dynamics near a folded node can combine with global features, leading to mixed mode oscillations. We review and extend the results of [26] on the dynamics near a folded node and state some results on mixed mode periodic orbits with Farey sequences of the form $1s$. We also show how to generalize the context of one fast variable to an arbitrary number of fast variables.

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Contributors: Brøns, M., Krupa, M., Wechselberger, M.
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Research output: Chapter in Book/Report/Conference proceeding » Book chapter – Annual report year: 2006 » Research » peer-review

**Streamline Patterns and their Bifurcations near a wall with Navier slip Boundary Conditions**
We consider the two-dimensional topology of streamlines near a surface where the Navier slip boundary condition applies. Using transformations to bring the streamfunction in a simple normal form, we obtain bifurcation diagrams of streamline patterns under variation of one or two external parameters. Topologically, these are identical with the ones previously found for no-slip surfaces. We use the theory to analyze the Stokes flow inside a circle, and show how it can be used to predict new bifurcation phenomena. ©2006 American Institute of Physics

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Publication date: 2006
Peer-reviewed: Yes

**Publication Information**
Journal: Physics of Fluids
Volume: 18
Issue number: 8
ISSN (Print): 1070-6631
Ratings:
Scopus rating (2006): SJR 2.12 SNIP 1.513
Web of Science (2006): Indexed yes
Original language: English
Electronic versions:
Brøns.pdf
DOIs:
10.1063/1.2337660
URLs:
http://link.aip.org/link/?PHFLE6/18/083102/1

**Bibliographical note**
Copyright (2006) American Institute of Physics. This article may be downloaded for personal use only. Any other use requires prior permission of the author and the American Institute of Physics.
Source: orbit
Source-ID: 192794
Research output: Contribution to journal » Journal article – Annual report year: 2006 » Research » peer-review

**Vortex Breakdown Generated by off-axis Bifurcation in a cylinder with rotating covers**
Vortex breakdown of bubble type is studied for the flow in a cylinder with rotating top and bottom covers. For large ratios of the angular velocities of the covers, we observe numerically that the vortex breakdown bubble in the steady regime may
occur through the creation of an off-axis vortex ring. This scenario does not occur in existing bifurcation theory based on a simple degeneracy in the flow field. We extend the theory to cover a non-simple degeneracy, and derive the associated bifurcation diagrams. We show that the vortex breakdown scenario involving a vortex ring can be explained from this theory, and that the numerically generated bifurcation diagrams are consistent with the theory.

General information
Publication status: Published
Organisations: Department of Mathematics, Dynamical systems, Fluid Mechanics, Department of Mechanical Engineering
Contributors: Bisgaard, A., Brøns, M., Sørensen, J. N.
Pages: 75-83
Publication date: 2006
Peer-reviewed: Yes

Publication information
Journal: Acta Mechanica
Volume: 187
ISSN (Print): 0001-5970
Ratings:
Scopus rating (2006): SJR 0.765 SNIP 0.962
Web of Science (2006): Indexed yes
Original language: English
DOIs: 10.1007/s00707-006-0367-y
Source-ID: 192887
Research output: Contribution to journal › Journal article – Annual report year: 2006 › Research › peer-review

Structures and Bifurcations In Fluid Flows with Applications to Vortex Breakdown and Wakes

General information
Publication status: Published
Organisations: Department of Mathematics, Dynamical systems
Contributors: Bisgaard, A. V., Brøns, M.
Publication date: Nov 2005

Publication information
Original language: English
Electronic versions:
Structure and Bifurcations in Fluid Flows.pdf
Source: orbit
Source-ID: 183859

Relaxation oscillations and canard in a nonlinear model of discontinuous plastic deformation in metals at very low temperature

General information
Publication status: Published
Organisations: Dynamical systems, Department of Mathematics
Contributors: Brøns, M.
Pages: 2289-2302
Publication date: 2005
Peer-reviewed: Yes

Publication information
Journal: Proceedings of the Royal Society of London, Series A
Volume: 461
Original language: English
Source: orbit
Source-ID: 182746
Research output: Contribution to journal › Journal article – Annual report year: 2005 › Research › peer-review
Streamline patterns and their bifurcations near a wall with Navier boundary conditions

General information
Publication status: Published
Organisations: Department of Mathematics, Technical University of Denmark
Contributors: Tophøj, L., Møller, S., Brøns, M.
Publication date: 2005

Publication information
Place of publication: Kongens Lyngby
Publisher: DCAMM, Technical University of Denmark
Original language: English
(DCAMM Report; No. 710).
URLs:
http://www.dcamm.dk
Source: orbit
Source-ID: 183855
Research output: Book/Report › Report – Annual report year: 2005 › Research › peer-review

Bifurcation of vortex breakdown patterns

General information
Publication status: Published
Organisations: Dynamical systems, Department of Mathematics
Contributors: Brøns, M., Bisgaard, A. V.
Pages: 988-993
Publication date: 2004

Host publication information
Title of host publication: Proceedings of the 2004 International Conference on Computational and Experimental Engineering and Sciences
Source: orbit
Source-ID: 132457
Research output: Chapter in Book/Report/Conference proceeding › Article in proceedings – Annual report year: 2004 › Research › peer-review

Relaxation oscillations and canards in a nonlinear model for discontinuous plastic deformation in metals at very low temperatures

General information
Publication status: Published
Organisations: Department of Mathematics
Contributors: Brøns, M.
Publication date: 2004

Publication information
Place of publication: Technical University of Denmark
Original language: English
(DCAMM Report; No. 700).
Source: orbit
Source-ID: 132456
Research output: Book/Report › Report – Annual report year: 2004 › Research › peer-review

Streamline topology of the near wake of a circular cylinder at low Reynolds numbers

General information
Publication status: Published
Organisations: Dynamical systems, Department of Mathematics, Roskilde University, HV Turbo
Contributors: Brøns, M., Niss, K., Jakobsen, B., Voigt, L. K., Bisgaard, A. V.
Publication date: 2004
Peer-reviewed: Yes
Event: Poster session presented at 21st International Congress of Theoretical and Applied Mechanics, Warsaw, Poland.
Source: orbit
Source-ID: 135414
Canard explosion in a model for discontinuous plastic deformation

General information
Publication status: Published
Organisations: Department of Mathematics
Contributors: Brøns, M.
Pages: 93-99
Publication date: 2003

Host publication information
Title of host publication: Proceedings of the 7th Conference on dynamical systems – theory and applications.
Place of publication: Lodz
Publisher: Department of Automatics and Biomechanics
ISBN (Print): 83-911746-5-4
Source: orbit
Source-ID: 28464
Research output: Chapter in Book/Report/Conference proceeding – Article in proceedings – Annual report year: 2003
Research: peer-review

Low-dimensional modeling of a driven cavity flow with two free parameters
By applying Proper Orthogonal Decomposition (POD) one is able to extract a limited amount of data which characterizes a flow of interest. The modes resulting from the decomposition form a basis in the phase space on which a Galerkin projection of the equations of motion can be performed. By carrying out such a procedure one obtains a low-dimensional model consisting of a reduced set of Ordinary Differential Equations (ODEs) which models the original equations. A technique called Sequential Proper Orthogonal Decomposition (SPOD) is developed to perform decompositions suitable for low-dimensional models. SPOD is capable of transforming data organized in different sets separately while still producing orthogonal modes. A low-dimensional model is constructed and used for analyzing bifurcations occurring in the flow in the lid-driven cavity with a rotating rod. The model allows one of the free parameters to appear in the inhomogeneous boundary conditions without the addition of any constraints. This is necessary because both the driving lid and the rotating rod are controlled simultaneously. Apparently, the results reported for this model are the first to be obtained for a low-dimensional model based on projections on POD modes for more than one free parameter.

General information
Publication status: Published
Organisations: Department of Mechanical Engineering, Department of Mathematics
Contributors: Jørgensen, B. H., Sørensen, J. N., Brøns, M.
Pages: 299-317
Publication date: 2003
Peer-reviewed: Yes

Publication information
Journal: Theoretical and Computational Fluid Dynamics
Volume: 16
Issue number: 4
ISSN (Print): 0935-4964
Ratings:
Scopus rating (2003): SJR 1.584 SNIP 0.918
Web of Science (2003): Indexed yes
Original language: English
DOIs:
10.1007/s00162-002-0082-9
Source: orbit
Source-ID: 28242
Research output: Contribution to journal › Journal article – Annual report year: 2003
Research: peer-review

Bifurcation to Infinity, Canards, and Chaotic Waves in Cubic Autocatalator Kinetics

General information
Publication status: Published
Organisations: Department of Mathematics
Contributors: Brøns, M., Sturis, J.
Explosion of limit cycles and chaotic waves in a simple nonlinear chemical system

A model of an autocatalytic chemical reaction was employed to study the explosion of limit cycles and chaotic waves in a nonlinear chemical system. The bifurcation point was determined using asymptotic analysis and perturbations. Scaling laws for amplitude and period were derived. A strong sensitivity was introduced due to bifurcation to infinity resulting in chaotic dynamics on adding diffusion.

General information
Publication status: Published
Organisations: Dynamical systems, Department of Mathematics, Novo Nordisk AS
Contributors: Brøns, M., Sturis, J.
Pages: 026209
Publication date: 2001
Peer-reviewed: Yes

Publication information
Volume: 64
Issue number: 2
ISSN (Print): 1063-651X
Ratings:
Scopus rating (2001): SJR 1.057 SNIP 1.393
Web of Science (2001): Indexed yes
Original language: English
Keywords: REACTION SCHEME

Electronic versions:
Brøns.pdf
DOIs:
10.1103/PhysRevE.64.026209
URLs:
http://link.aps.org/doi/10.1103/PhysRevE.64.026209
Topology of vortex Breakdown bubbles in a cylinder with rotating bottom and Free surface
The flow patterns in the steady, viscous flow in a cylinder with a rotating bottom and a free surface are investigated by a combination of topological and numerical methods. Assuming the flow is axisymmetric, we derive a list of possible bifurcations of streamline structures on varying two parameters, the Reynolds number and the aspect ratio of the cylinder. Using this theory we systematically perform numerical simulations to obtain the bifurcation diagram. The stability limit for steady flow is found and established as a Hopf bifurcation. We compare with the experiments by Spohn, Mory & Hopfinger (1993) and find both similarities and differences.

Evaluation of POD-based decomposition techniques applied to parameter-dependent non-turbulent flows

Streamline topologies near simple degenerate critical points in two-dimensional flow away from boundaries
Streamline patterns and their bifurcations in two-dimensional incompressible flow are investigated from a topological point of view. The velocity field is expanded at a point in the fluid, and the expansion coefficients are considered as bifurcation parameters. A series of nonlinear coordinate changes results in a much simplified system of differential equations for the streamlines (a normal form) encapsulating all the features of the original system. From this, we obtain a complete description of bifurcations up to codimension three close to a simple linear degeneracy. As a special case we develop the theory for flows with reflectional symmetry. We show that all the patterns obtained can be realized in steady Navier–Stokes or Stokes flow, but an unresolved difficulty arises in the symmetric case for Navier–Stokes flow. The theory is applied to the patterns and bifurcations found numerically in two recent studies of Stokes flow in confined domains. ©1999 American Institute of Physics.
Streamline topology of steady axisymmetric vortex breakdown in a cylinder with co- and counter-rotating end-covers

Using a combination of bifurcation theory for two-dimensional dynamical systems and numerical simulations, we systematically determine the possible flow topologies of the steady vortex breakdown in axisymmetric flow in a cylindrical container with rotating end-covers. For fixed values of the ratio of the angular velocities of the covers in the range from -0.02 to 0.05, bifurcations of recirculating bubbles under variation of the aspect ratio of the cylinder and the Reynolds number are found. Bifurcation curves are determined by a simple fitting procedure of the data from the simulations. For the much studied case of zero rotation ratio (one fixed cover) a complete bifurcation diagram is constructed. Very good agreement with experimental results is obtained, and hitherto unresolved details are determined in the parameter region where up to three bubbles exist. For non-zero rotation ratios the bifurcation diagrams are found to change dramatically and give rise to other types of bifurcations.
Evaluation of POD-based decomposition techniques applied to parameter-dependent non-turbulent flows

General information
Publication status: Published
Organisations: Department of Mathematics, New York University
Contributors: Christensen, E. A., Brøns, M., Sørensen, J. N.
Publication date: 1998

Publication information
Original language: English
Source: orbit
Source-ID: 167751

On stagnation points and streamline topology in vortex flows
The problem of locating stagnation points in the flow produced by a system of N vortices in two dimensions is considered. The general solution follows from a 1864 theorem by Siebeck, that the stagnation points are the foci of a certain plane curve of class N-1 that has all lines connecting vortices pairwise as tangents. The case N=3, for which Siebeck's curve is a conic, is considered in some detail. It is shown that the classification of the type of conic coincides with the known classification of regimes of motion for the three vortices. A similarity result for the triangular coordinates of the stagnation point in a flow produced by three vortices with sum of strengths zero is found. Using topological arguments the distinct streamline patterns for flow about three vortices are also determined. Partial results are given for two special sets of vortex strengths on the changes between these patterns as the motion evolves. The analysis requires a number of unfamiliar mathematical tools which are explained.

General information
Publication status: Published
Organisations: Dynamical systems, Department of Mathematics, University of Illinois
Contributors: Aref, H., Brøns, M.
Pages: 1-27
Publication date: 1998
Peer-reviewed: Yes

Publication information
Journal: Journal of Fluid Mechanics
Volume: 370
ISSN (Print): 0022-1120
Original language: English
DOIs: 10.1017/S0022112098001761
Source: orbit
Source-ID: 169898
Research output: Contribution to journal > Journal article – Annual report year: 1998 > Research > peer-review

Streamline topologies near simple degenerate critical points in two-dimensional flow away from boundaries
Streamline patterns and their bifurcations in two-dimensional incompressible flow are investigated from a topological point of view. The velocity field is expanded at a point in the fluid, and the expansion coefficients are considered as bifurcation parameters. A series of non-linear coordinate changes results in a much simplified system of differential equations for the streamlines (a normal form) encapsulating all the features of the original system. From this, we obtain a complete description of bifurcations up to codimension three close to a simple linear degeneracy. As a special case we develop the theory for flows with reflectional symmetry. We show that all the patterns obtained can be realized in steady Navier-Stokes or Stokes flow, but an unresolved difficulty arises in the symmetric case for Navier-Stokes flow. The theory is applied to the patterns and bifurcations found numerically in two recent studies of Stokes flow in confined domains.

General information
Publication status: Published
Organisations: Department of Mathematics
Contributors: Brøns, M., Hartnack, J. N.
Publication date: 1998

Publication information
Original language: English
Source: orbit
Source-ID: 170156
Streamline topology of steady axisymmetric vortex breakdown in a cylinder with co- and counter-rotating end-covers

General information
Publication status: Published
Organisations: Department of Mathematics
Contributors: Brøns, M., Voigt, L. P. K., Sørensen, J. N.
Publication date: 1998

Publication information
Original language: English
Source: orbit
Source-ID: 169899
Research output: Book/Report › Report – Annual report year: 1998 › Research › peer-review

Topological fluid mechanics of Axisymmetric Flow
Topological fluid mechanics in the sense of the present paper is the study and classification of flow patterns close to a critical point. Here we discuss the topology of steady viscous incompressible axisymmetric flows in the vicinity of the axis. Following previous studies the velocity field $v$ is expanded in a Taylor series at a point on the axis, and the expansion coefficients are considered as bifurcation parameters. After a normal form transformation we easily obtain the most common bifurcations of the flow patterns. The use of non-linear normal forms provide a gross simplification, which to the authors knowledge has not been used systematically to high orders in topological fluid mechanics. We compare the general results with experimental and computational results on the Vogel-Ronneberg flow. We show that the topology changes observed when recirculating bubbles on the vortex axis are created and interact follow the topological classification and that the complete set of patterns found is contained in a codimension-4 unfolding of the most simple singular configuration.

General information
Publication status: Published
Organisations: Department of Mathematics
Contributors: Brøns, M.
Pages: 213-222
Publication date: 1998

Host publication information
Title of host publication: Simulation and Identification of Organized Structures in Flows
Place of publication: Dordrecht, NL
Publisher: Kluwer Academic Publishers
Source: orbit
Source-ID: 169900
Research output: Chapter in Book/Report/Conference proceeding › Article in proceedings – Annual report year: 1998 › Research › peer-review

Circle maps and the Devil's staircase in a periodically perturbed Oregonator
Markman and Bar-Eli has studied a periodically forced Oregonator numerically and found a parameter range with the following properties: (1) Only periodic solutions are found in frequency-locked steps, each with a certain pattern of large and small oscillations (2) Between any two steps there is a step with the period being the sum of the two periods and the concatenation of the two patterns (3) Certain scaling properties as the period tends to infinity. We show that such behavior occurs if the dynamics of the system is governed by a family of diffeomorphisms of a circle with a Devil's staircase. Using invariant manifold theory we argue that an invariant circle must indeed exist when, as in the present case, the unforced system is close to a saddle-loop bifurcation. Generalisations of the results are briefly discussed.

General information
Publication status: Published
Organisations: Dynamical systems, Department of Mathematics, Aalborg University, Tel Aviv University
Contributors: Brøns, M., Gross, P., Bar-Eli, K.
Pages: 2621-2628
Publication date: 1997
Peer-reviewed: Yes

Publication information
Volume: 7
Issue number: 11
Local and global bifurcations at infinity in models of glycolytic oscillations

We investigate two models of glycolytic oscillations. Each model consists of two coupled nonlinear ordinary differential equations. Both models are found to have a saddle point at infinity and to exhibit a saddle-node bifurcation at infinity, giving rise to a second saddle and a stable node at infinity. Depending on model parameters, a stable limit cycle may blow up to infinite period and amplitude and disappear in the bifurcation, and after the bifurcation, the stable node at infinity then attracts all trajectories. Alternatively, the stable node at infinity may coexist with either a stable sink (not at infinity) or a stable limit cycle. This limit cycle may then disappear in a heteroclinic bifurcation at infinity in which the unstable manifold from one saddle at infinity joins the stable manifold of the other saddle at infinity. These results explain prior reports for one of the models concerning parameter values for which the system does not admit any physical (bounded) behavior. Analytic results on the scaling of amplitude and period close to the bifurcations are obtained and confirmed by numerical computations. Finally, we consider more realistic modified models where all solutions are bounded and show that some of the features stemming from the bifurcations at infinity are still present.

On Stagnation points and streamline topology in vortex flows

The problem of locating stagnation points in the flow produced by a system of N interacting point vortices in two dimensions is considered. The general solution, which follows from an 1864 theorem by Siebeck, that the stagnation points are the foci of a certain plane curve of class N-1 that has all lines connecting vortices pairwise as tangents, is stated and proved. Specific results for the case N=3 are proved. The related problem of the location of stagnation points in a frame of reference moving with the vortices, when these are translating uniformly, is considered and an extension of Siebeck’s theorem to this case is stated.

Progress in Industrial Mathematics at ECMI 96

Proceedings of the 9th conference of the European Consortium for Mathematics in Industry, ECMI 96, Technical University of Denmark. The use of mathematical models in industry is steadily growing. There is a continuous need in industry for new and efficient mathematical techniques, and university mathematicians get inspiration from industrial demands. The European Consortium for Mathematics in Industry aims to create contact between industry and academia, and to promote research in industrial mathematics. This book contains a broad spectrum of mathematics applied to
industrial problems. Applied mathematics, case studies, and review papers in the following fields are included: Environmental modelling, railway systems, industrial processes, electronics, ships, oil industry, optimization, machine dynamics, fluids in industry. Applied mathematicians and other professionals working in academia or industry will find the book to be useful and inspiring source of mathematical applications related to industrial problems.

General information
Publication status: Published
Organisations: Department of Mathematics
Publication date: 1997

Publication information
Place of publication: Stuttgart
Publisher: B.G. Teubner Stuttgart Leipzig
Original language: English
Source: orbit
Source-ID: 166613
Research output: Book/Report › Book – Annual report year: 1997 › Research › peer-review

Streamline topology of axisymmetric flows

General information
Publication status: Published
Organisations: Department of Mathematics
Contributors: Brøns, M.
Publication date: 1997

Publication information
Publisher: MAT, DTU
Original language: English
Source: orbit
Source-ID: 166856
Research output: Book/Report › Report – Annual report year: 1997 › Research › peer-review

Circle Maps and the Devil's Staircase in a Chemical Oscillator
We explain numerical results on a periodically perturbed Oregonator by Markman and Bar-Eli (J. Phys. Chem. 98 12248 (1994)). If the dynamics of the system is governed by a family of diffeomorphisms of a circle with a Devil's staircase one will expect the observed behavior, i.e. (1) Only periodic solutions are found in frequency-locked steps, each with some pattern of large and small oscillations (2) Between any two steps there is a step with the period being the sum of the periods and the concatenated pattern (3) Scaling as the period tends to infinity. Using invariant manifold theory we argue that an invariant circle must indeed exist when, as in the case under consideration, the unperturbed system is close to a saddle-node bifurcation on a limit cycle. Generalisations of the results are discussed.

General information
Publication status: Published
Organisations: Department of Mathematics, Aalborg University, Tel Aviv University
Contributors: Brøns, M., Gross, P., Bar-Eli, K.
Number of pages: 10
Publication date: 1996

Publication information
Original language: English
Source: orbit
Source-ID: 165816
Research output: Book/Report › Report – Annual report year: 1996 › Research › peer-review

Coriolis Effects in the Dynamics of a Rotating Elastic Structure
Small oscillations of a rotating elasticum with a mass at the free end are investigated with Poincare-Lindstedt series. It is shown that the mass moves on a figure-eight shaped curve in a direction determined by the sign of the angular velocity and hence that the Coriolis force influences the motion.

General information
Publication status: Published
Organisations: Department of Mathematics
Dynamics of a Simple Rotating Structure
Small oscillations of an elasticum whose one end is constrained to move in uniform circular motion and whose free end carries a mass, are investigated with Poincare-Lindstedt series. It is demonstrated that the mass traverses a figure-eight shaped curve in a direction determined by the sign of the angular velocity and hence that the Coriolis force does influence the motion.

Introduction to Dynamics, Supplementary Notes 0211/1743

Topological Fluid Mechanics of Axisymmetric Flows
Topological Fluid Mechanics of Axisymmetric Flows

General information
Publication status: Published
Organisations: Department of Mathematics
Contributors: Brøns, M.
Pages: 61-63
Publication date: 1996

Topological Fluid Mechanics with Applications to Free Surfaces and Axisymmetric Flows
Topological fluid mechanics is the study of qualitative features of fluid patterns. We discuss applications to the flow beneath a stagnant surface film, and to patterns in axisymmetric flow.

General information
Publication status: Published
Organisations: Department of Mathematics
Contributors: Brøns, M.
Pages: 73-74
Publication date: 1996
Peer-reviewed: Yes

Topological fluid dynamics of interfacial flows
The topological description of flows in the vicinity of a solid boundary, that is familiar from the aerodynamics literature, has recently been extended to the case of flow at a liquid–gas interface or a free surface by Lugt [Phys. Fluids 30, 3647 (1987)]. Lugt’s work is revisited in a more general setting, including nonconstant curvature of the interface and gradients of surface tension, using tools of modern nonlinear dynamics. Bifurcations of the flow pattern occur at degenerate configurations. Using the theory of unfolding, this paper gives a complete description of the bifurcations that depend on terms up to the second order. The general theory of this paper is applied to the topology of streamlines during the breaking of a wave and to the flow below a stagnant surface film. Physics of Fluids is copyrighted by The American Institute of Physics.

General information
Publication status: Published
Organisations: Dynamical systems, Department of Mathematics
Contributors: Brøns, M.
Pages: 2730-2737
Projects:

On the Accuracy in Measurements of Time Intervals and Traffic Intensities with Application to Teletraffic and Simulation.
Iversen, V. B., PhD Student, Department of Mathematics
Brøns, M., Main Supervisor
01/04/1968 → 15/12/1976
Project: PhD

Operations Research in Production Planning - Interconnections between Production and Demand.
Vidal, R. V. V., PhD Student, Department of Mathematics
Brøns, M., Main Supervisor
01/02/1969 → 28/05/1971
Project: PhD

Methods for Control of Complex Dynamics Systems.
Drud, A., PhD Student, Department of Mathematics
Brøns, M., Main Supervisor
01/08/1973 → 20/04/1977
Project: PhD

ECOSYSTEMS - An Operational Research Approach
Beyer, J., PhD Student, Department of Mathematics
Brøns, M., Main Supervisor
15/12/1976 → 15/12/1976
Project: PhD

Topology of Exotic Wakes
Nielsen, A. R., PhD Student, Department of Mathematics
Brøns, M., Main Supervisor
Heil, M., Supervisor
Forskningsrådsfinansiering
01/09/2017 → 31/08/2020
Award relations: Topology of Exotic Wakes
Project: PhD
Hvirveldynamik og strømningstopologi
Andersen, M., PhD Student, Department of Informatics and Mathematical Modeling
Brøns, M., Main Supervisor
Bohr, T., Examiner
Kanso, E., Examiner
Stremler, M. A., Examiner
Technical University of Denmark
01/08/2010 → 22/11/2013
Award relations: Hvirveldynamik og strømningstopologi
Project: PhD

Topologisk Fluid Dynamik: Bifurkationer og strukturer i strømninger
Bisgaard, A. V., PhD Student, Department of Mathematics
Brøns, M., Main Supervisor
Sørensen, J. N., Supervisor
Sørensen, M. P., Examiner
Krupa, M., Examiner, Department of Applied Mathematics and Computer Science
Ottesen, J. T., Examiner
Krupa, M., Examiner
DTU-lønnet stipendie
01/02/2002 → 02/01/2006
Award relations: Topologisk Fluid Dynamik: Bifurkationer og strukturer i strømninger
Project: PhD

Hæmodynamisk modellering af hjertet
Adeler, P. T., PhD Student, Department of Informatics and Mathematical Modeling
Barker, V. A., Main Supervisor
Larsen, J. K., Supervisor
Thomsen, P. G., Supervisor
Brøns, M., Examiner
Verdonck, P., Examiner
Grisen, G., Examiner
Erhvervsforskerordningen
01/08/1997 → 15/01/2002
Award relations: Hæmodynamisk modellering af hjertet
Project: PhD

Simulering og identifikation af strukturer i komplexe fluidstrømninger
Jørgensen, B. H., PhD Student, Department of Mechanical Engineering
Sørensen, J. N., Main Supervisor
Brøns, M., Supervisor
Larsen, R., Supervisor
Meyer, K. E., Examiner
Veldman, A. E. P., Examiner
Mann, J., Examiner
DTU stipendium
01/09/1996 → 26/03/2000
Award relations: Simulering og identifikation af strukturer i komplexe fluidstrømninger
Project: PhD

Bedømmelse af den atmosfæriske turbulens over områder af varierende overfladetype
Falk, A. K. V., PhD Student, Department of Mathematics
Brøns, M., Main Supervisor
Ditlevsen, O. D., Examiner
Heimburg, T., Examiner
Forskerakademiets Samfinansier
01/01/1996 → 08/07/1999
Award relations: Bedømmelse af den atmosfæriske turbulens over områder af varierende overfladetype
Project: PhD

Selv-organiserede processer og kohærente strukturer i kontinuumssystemer
Schmidt, M. R., PhD Student, Department of Informatics and Mathematical Modeling
Bifurkationsstrukturer i koblede ikke-lineære oscillatorer med særlig henblik på biologiske systemer

Knudsen, C., PhD Student, Department of Physics
Mosékilde, E., Main Supervisor
Brøns, M., Supervisor
Bohr, T., Examiner

Gammel ordning u/skema-SU
01/08/1991 → 17/06/1994

Award relations: Bifurkationsstrukturer i koblede ikke-lineære oscillatorer med særlig henblik på biologiske systemer
Project: PhD

Laminar-turbulent omslag i roterende strømning

Christensen, E. A., PhD Student
Brøns, M., Supervisor
Larsen, P. S., Supervisor
Serensen, J. N., Supervisor

Gammel ordning u/skema-SU
01/07/1991 → 30/05/1994

Award relations: Laminar-turbulent omslag i roterende strømning
Project: PhD

From non-smooth to smooth - on regularization using slow-fast theory

Bossolini, E., PhD Student, Department of Applied Mathematics and Computer Science
Brøns, M., Main Supervisor, Department of Applied Mathematics and Computer Science
Galvanetto, U., Supervisor
Kristiansen, K. U., Supervisor, Department of Applied Mathematics and Computer Science
Henriksen, C., Examiner, Department of Applied Mathematics and Computer Science
Hogan, S. J., Examiner
Szmolyan, P., Examiner

Technical University of Denmark
15/10/2014 → 13/12/2017

Award relations: From non-smooth to smooth - on regularization using slow-fast theory
Project: PhD

Aircraft, Bifurcation and Control

Cromme, M., PhD Student, Department of Mathematics
Brøns, M., Main Supervisor
Stoustrup, J., Supervisor
Jørgensen, S. B., Examiner
Perram-John, W., Examiner

DTU stipendium
01/12/1994 → 13/10/1998

Award relations: Aircraft, Bifurcation and Control
Project: PhD

Endelig-dimensionale dynamiske systemer i fluid mekanik

Hartnack, J. N., PhD Student, Department of Mathematics
Brøns, M., Main Supervisor

DTU stipendium
01/09/1996 → 29/09/1999

Award relations: Endelig-dimensionale dynamiske systemer i fluid mekanik
Project: PhD
Nonlinear Fluid
Jensen, J. S., PhD Student, Department of Mechanical Engineering
Thomsen, J. J., Main Supervisor
Blekhman, I., Examiner
Brøns, M., Examiner
Olhoff, N., Examiner
DTU stipendium
01/02/1996 → 21/06/1999
Award relations: Nonlinear Fluid -
Project: PhD

Osmotically driven flows in micro and nanofluidic systems and their applications to sugar transport in plants
Jensen, K. H., PhD Student, Department of Micro- and Nanotechnology
Bruus, H., Main Supervisor
Bohr, T., Supervisor
Brøns, M., Examiner
Eijkel, J. C. T., Examiner
Schulz, A., Examiner
DTU, Samfinansiering
01/08/2007 → 20/04/2011
Award relations: Osmotically driven flows in micro and nanofluidic systems and their applications to sugar transport in plants
Project: PhD

Fluid Dynamics of Animal Locomotion
Schnipper, T., PhD Student, Department of Physics
Andersen, A. P., Main Supervisor
Aref, H., Supervisor
Bohr, T., Supervisor
Sørensen, J. N., Supervisor
Brøns, M., Examiner
Water, W. V. D., Examiner
Williamson, C. H. K., Examiner
DTU, Samfinansiering
15/08/2007 → 02/02/2011
Award relations: Fluid Dynamics of Animal Locomotion
Project: PhD

Design af digitale optiske filtre
Mikkelsen, C. I., PhD Student, Department of Micro- and Nanotechnology
Bruus, H., Main Supervisor
Hansen, M. F., Supervisor
Brøns, M., Examiner
Rasmussen, J. J., Examiner
Schönfeld, F., Examiner
DTU-lønnet stipendie
01/02/2001 → 29/08/2005
Award relations: Design af digitale optiske filtre
Project: PhD

Topology Optimization in Micro- and Nanofluidic Systems
Gregersen, M. M., PhD Student, Department of Micro- and Nanotechnology
Bruus, H., Main Supervisor
Hansen, O., Supervisor
Brøns, M., Examiner
Hardt, S., Examiner
Ramos Reyes, A., Examiner
DTU-lønnet stipendie
01/01/2006 → 20/05/2009
Award relations: Topology Optimization in Micro- and Nanofluidic Systems
Project: PhD
Topological fluid dynamics: Symmetry breaking and fluid interfaces
Balci, A., PhD Student, Department of Mathematics
Brøns, M., Main Supervisor
Henriksen, C., Examiner
Hartnack, J. N., Examiner
Blackmore, D., Examiner
Stipendie fra udlandet
01/09/2012 → 19/11/2015
Award relations: Topological fluid dynamics: Symmetry breaking and fluid interfaces
Project: PhD

Matematisk modellering af membransepation
Vinther, F., PhD Student, Department of Informatics and Mathematical Modeling
Brøns, M., Main Supervisor
Meyer, A. S., Supervisor
Sørensen, M. P., Supervisor
Hassager, O., Examiner
Jönsson, A., Examiner
Davis, R. H., Examiner
Technical University of Denmark
01/04/2010 → 20/03/2014
Award relations: Matematisk modellering af membransepation
Project: PhD

Analysis of Body-Vortex Interactions
Pedersen, J. R., PhD Student, Department of Mathematics
Brøns, M., Main Supervisor
Aref, H., Supervisor
Bohr, T., Examiner
Eckhardt, B., Examiner
Williamson, C. H. K., Examiner
Technical University of Denmark
01/01/2008 → 30/03/2011
Award relations: Analysis of Body-Vortex Interactions
Project: PhD

Dynamical Systems Approach to L-H Transition in Magnetically Confined Plasma
Dam, M., PhD Student, Department of Mathematics
Brøns, M., Main Supervisor
Naulin, V., Supervisor
Rasmussen, J. J., Supervisor
Sørensen, M. P., Examiner
Garcia, O. E., Examiner
Jensen, M. H., Examiner
Technical University of Denmark
15/12/2013 → 17/01/2018
Award relations: Dynamical Systems Approach to L-H Transition in Magnetically Confined Plasma
Project: PhD

Homoclinic Bifurcation to Infinity
This project is concerned with investigations of the homoclinic bifurcation to infinity for spatially distributed systems. The resulting mathematical models are partial differential equations. Specific models of importance for biochemistry are investigated.
Brøns, M., Project Manager, Department of Mathematics
Sturis, J., Project Participant, Novo Nordisk AS
01/01/1996 → …
Collaborators: Novo Nordisk AS
Project: Research
Reduction methods for Partial Differential Equations
Rotating flows connected with vortex breakdown are investigated with proper orthogonal decomposition (POD), based on a numerical solution of the Navier-Stokes equations.
Brøns, M., Project Manager, Department of Mathematics
Serensen, J. N., Project Participant, Department of Energy Engineering
Christensen, E. A., Project Participant, City College of New York
01/01/1996 → …
Collaborators: City College of New York
Project: Research

Bifurcation in Chemical Systems
Bifurcations in systems of chemical reactions are investigated with a combination of analytical and numerical methods.
Brøns, M., Project Manager, Department of Mathematics
Bar-Eli, K., Project Participant, Tel Aviv University
01/01/1996 → …
Collaborators: Tel Aviv University
Project: Research

Stagnation Points in Point Vortex Flows
This project is concerned with a description of the stagnation points and the streamline topology in a flow generated by few point vortices. Some classical results from algebraic geometry find a surprising application here.
Brøns, M., Project Manager, Department of Mathematics
Aref, H., Project Participant, University of Illinois at Urbana-Champaign
01/01/1996 → …
Collaborators: University of Illinois at Urbana-Champaign
Project: Research

Three-dimensional topology of the vortex breakdown
The vortex breakdown is the creation of a secondary flow structure around a vortex. Due to its occurrence in many flows in technology and nature, this is a very active research area. The present project is concerned with some recently discovered three-dimensional effects which has important bearing on experimental visualisation of the flow. The project attempts to use bifurcation theory to qualitatively explain and predict the experimental results.
Brøns, M., Project Manager, Department of Mathematics
Spohn, A., Project Participant, Universite de Poitiers
01/08/1997 → …
Collaborators: University of Poitiers, Universite de Poitiers
Project: Research

BIFD2006: Second International Symposium on Bifurcations and Instabilities in Fluid Dynamics
Hydrodynamic stability is of fundamental importance in fluid dynamics and is a well-established subject of scientific investigation that continues to attract great interest of the fluid mechanics community. Hydrodynamic instabilities of prototypical character are, for example, the Rayleigh-Bénard, the Taylor-Couette, the Bénard-Marangoni, the Rayleigh-Taylor, and the Kelvin-Helmholtz instabilities. A fundamental understanding of various patterns of bifurcations such as identifying the most dominant mechanisms responsible for the instability threshold is also required if one is to design reliable and efficient industrial processes and applications, such as melting, mixing, crystal growth, coating, welding, flow re-attachment over wings, and others.
The symposium aimed at bringing together scholars with mutual interest in computational, experimental, and theoretical methods for the analysis of bifurcation and instability phenomena in fluid dynamics.
Brøns, M., Project Manager, Department of Mathematics
Serensen, J. N., Project Participant, Department of Mathematics
Forskningsrådene - STVF: DKK75,000.00
01/08/2006 → 31/03/2007
Award relations: Second International Symposium on Bifurcations and Instabilities in Fluid Dynamics
Project: Research

Applied topological fluid mechanics: Bioreactors and bluff body wakes
Brøns, M., Project Manager
Topological Fluid Mechanics
The project is concerned with a qualitative description of flow patterns, mainly in two dimensions. The aim is to classify possible patterns and their bifurcations as external parameters are varied. The results are applied to viscous and Stokes flows and flows with different types of symmetry.

Brøns, M., Project Manager, Department of Mathematics
Hartnack, J. N., Project Participant, Department of Mathematics
01/01/1997 → …
Project: Research

Dynamics of Plane Euler Elastica
The configuration space of plane Euler elastica is a two dimensional space with a simple potential function. The associated dynamical system is studied in order to describe the movement of the free end.

Gravesen, J., Project Manager, Department of Mathematics
Brøns, M., Project Participant, Department of Mathematics
Hjorth, P. G., Project Participant, Department of Mathematics
Markvorsen, S., Project Participant, Department of Mathematics
Sinclair, R., Project Participant, Department of Mathematics
01/09/1997 → …
Project: Research

European Consortium for Mathematics in Industry (ECMI)
The goal of ECMI is to promote the interaction between universities and research groups in industry. The aims of ECMI are to promote the use of mathematical models in industry, to educate industrial mathematicians to meet the growing demand for such experts and finally to operate on a European scale. Activities comprises: Attracting EU funding for post docs and research in industrial mathematics, student exchange through ERASMUS, organizing ECMI modelling weeks, biannual conferences, European Study Groups for Industry (ESGI), and others.

Sørensen, M. P., Project Manager, Department of Informatics and Mathematical Modeling
Poulsen, N. K., Project Participant, Department of Informatics and Mathematical Modeling
Brøns, M., Project Participant, Department of Mathematics
Hjorth, P. G., Project Participant, Department of Mathematics
Skovgaard, O., Project Participant, Department of Mathematics
Christensen, O., Project Participant, Department of Mathematics
01/01/1991 → …
Project: Research

Activities:

SIAM Conference on Applications of Dynamical Systems 2017
Period: 22 May 2017
Morten Brøns (Participant)
Kristian Uldall Kristiansen (Participant)
Alan R. Champneys (Chairman)
John Hogan (Chairman)
Department of Applied Mathematics and Computer Science
Mathematics

**Description**
Canards in Stiction: On Solutions of a Friction Oscillator by Regularization

We consider the problem of the friction oscillator using the stiction model of friction. This friction law has a discontinuity between the dynamic and the static regime. The discontinuity set has a sticking region in which the forward solution is non-unique. In particular, there are special points along these segments where the solution is tangent to the boundary of the discontinuity set. In order to resolve this uncertainty, we introduce a regularization of the vector field and we obtain a multiple-time scale problem. Here the special points of the piecewise-smooth problem become folded saddles and a canard solution appears. We study the interaction of periodic orbits with the canard and we find that the the regularized problem has solutions that do not appear in the original problem.

Degree of recognition: **International**

Links:
http://meetings.siam.org/sess/dsp_programsess.cfm?SESSIONCODE=61861 (Minisymposium description)

**Related event**
**SIAM Conference on Applications of Dynamical Systems 2017**
21/05/2017 – 26/05/2017
Snowbird, United States
Activity: Attending an event › Participating in or organising a conference