Gabor Frames in $\ell^2(\mathbb{Z})$ and Linear Dependence

We prove that an overcomplete Gabor frame in $(\ell^2(\mathbb{Z}))$ generated by a finitely supported sequence is always linearly dependent. This is a particular case of a general result about linear dependence versus independence for Gabor systems in $(\ell^2(\mathbb{Z}))$ with modulation parameter $1/M$ and translation parameter $N$ for some $(\ell^2(\mathbb{Z}))$ and generated by a finite sequence $g$ in $(\ell^2(\mathbb{Z}))$ with $K$ nonzero entries.
Operator representations of frames

The purpose of this paper is to consider representations of frames \( \{f_k\}_{k \in I} \) in a Hilbert space \( \mathcal{H} \) of the form \( \{f_k\}_{k \in I} = \{T f_0\}_{k \in I} \) for a linear operator \( T \); here the index set \( I \) is either \( \mathbb{Z} \) or \( \mathcal{L}_0 \). While a representation of this form is available under weak conditions on the frame, the analysis of the properties of the operator \( T \) requires more work. For example it is a delicate issue to obtain a representation with a bounded operator, and the availability of such a representation not only depends on the frame considered as a set, but also on the chosen indexing. Using results from operator theory we show that by embedding the Hilbert space \( \mathcal{H} \) into a larger Hilbert space, we can always represent a frame via iterations of a bounded operator, composed with the orthogonal projection onto \( \mathcal{H} \). The paper closes with a discussion of an open problem concerning representations of Gabor frames via iterations of a bounded operator.

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Operator Representations of Frames: Boundedness, Duality, and Stability

The purpose of the paper is to analyze frames (Formula presented.) having the form (Formula presented.) for some linear operator (Formula presented.). A key result characterizes boundedness of the operator \( T \) in terms of shift-invariance of a certain sequence space. One of the consequences is a characterization of the case where the representation (Formula presented.) can be achieved for an operator \( T \) that has an extension to a bounded bijective operator (Formula presented.). In this case we also characterize all the dual frames that are representable in terms of iterations of an operator \( V \); in particular we prove that the only possible operator is (Formula presented.) Finally, we consider stability of the
representation (Formula presented.) rather surprisingly, it turns out that the possibility to represent a frame on this form is sensitive towards some of the classical perturbation conditions in frame theory. Various ways of avoiding this problem will be discussed. Throughout the paper the results will be connected with the operators and function systems appearing in applied harmonic analysis, as well as with general group representations.

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Explicit constructions and properties of generalized shift-invariant systems in $L^2(\mathbb{R})$

Generalized shift-invariant (GSI) systems, originally introduced by Hernández et al. and Ron and Shen, provide a common framework for analysis of Gabor systems, wavelet systems, wave packet systems, and other types of structured function systems. In this paper we analyze three important aspects of such systems. First, in contrast to the known cases of Gabor frames and wavelet frames, we show that for a GSI system forming a frame, the Calderón sum is not necessarily bounded by the lower frame bound. We identify a technical condition implying that the Calderón sum is bounded by the lower frame bound and show that under a weak assumption the condition is equivalent with the local integrability condition introduced by Hernández et al. Second, we provide explicit and general constructions of frames and dual pairs of frames having the GSI-structure. In particular, the setup applies to wave packet systems and in contrast to the constructions in the literature, these constructions are not based on characteristic functions in the Fourier domain. Third, our results provide insight into the local integrability condition (LIC).

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