A proof of the Barát-Thomassen conjecture

The Barát-Thomassen conjecture asserts that for every tree $T$ on $m$ edges, there exists a constant $k_T$ such that every $k_T$-edge-connected graph with size divisible by $m$ can be edge-decomposed into copies of $T$. So far this conjecture has only been verified when $T$ is a path or when $T$ has diameter at most 4. Here we prove the full statement of the conjecture.
Decomposing graphs into a constant number of locally irregular subgraphs

A graph is locally irregular if no two adjacent vertices have the same degree. The irregular chromatic index \( \chi\text{irr}'(G) \) of a graph \( G \) is the smallest number of locally irregular subgraphs needed to edge-decompose \( G \). Not all graphs have such a decomposition, but Baudon, Bensmail, Przybyło, and Woźniak conjectured that if \( G \) can be decomposed into locally irregular subgraphs, then \( \chi\text{irr}'(G) \leq 3 \). In support of this conjecture, Przybyło showed that \( \chi\text{irr}'(G) \leq 3 \) holds whenever \( G \) has minimum degree at least 1010.

Here we prove that every bipartite graph \( G \) which is not an odd length path satisfies \( \chi\text{irr}'(G) \leq 10 \). This is the first general constant upper bound on the irregular chromatic index of bipartite graphs. Combining this result with Przybyło’s result, we show that \( \chi\text{irr}'(G) \leq 328 \) for every graph \( G \) which admits a decomposition into locally irregular subgraphs. Finally, we show that \( \chi\text{irr}'(G) \leq 2 \) for every 16-edge-connected bipartite graph \( G \).

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Decomposing highly edge-connected graphs into homomorphic copies of a fixed tree

The Tree Decomposition Conjecture by Barát and Thomassen states that for every tree \( T \) there exists a natural number \( k(T) \) such that the following holds: If \( G \) is a \( k(T) \)-edge-connected simple graph with size divisible by the size of \( T \), then \( G \) can be edge-decomposed into subgraphs isomorphic to \( T \). So far this conjecture has only been verified for paths, stars, and a family of bistars. We prove a weaker version of the Tree Decomposition Conjecture, where we require the subgraphs in the decomposition to be isomorphic to graphs that can be obtained from \( T \) by vertex-identifications. We call such a subgraph a homomorphic copy of \( T \). This implies the Tree Decomposition Conjecture under the additional constraint that the girth of \( G \) is greater than the diameter of \( T \). As an application, we verify the Tree Decomposition Conjecture for all trees of diameter at most 4.
Graph Decompositions

The topic of this PhD thesis is graph decompositions. While there exist various kinds of decompositions, this thesis focuses on three problems concerning edgedecompositions. Given a family of graphs $H$ we ask the following question: When can the edge-set of a graph be partitioned so that each part induces a subgraph isomorphic to a member of $H$? Such a decomposition is called an $H$-decomposition. Apart from the existence of an $H$-decomposition, we are also interested in the number of parts needed in an $H$-decomposition.

Firstly, we show that for every tree $T$ there exists a constant $k(T)$ such that every $k(T)$-edge-connected graph whose size is divisible by the size of $T$ admits a $T$-decomposition. This proves a conjecture by Barát and Thomassen from 2006.

Moreover, we introduce a new arboricity notion where we restrict the diameter of the trees in a decomposition into forests. We conjecture that for every natural number $k$ there exists a natural number $d(k)$ such that the following holds: If $G$ can be decomposed into $k$ forests, then $G$ can be decomposed into $k + 1$ forests in which each tree has diameter at most $d(k)$. We verify this conjecture for $k \leq 3$. As an application we show that every 6-edge-connected planar graph contains two edge-disjoint 18/19-thin spanning trees.

Finally, we make progress on a conjecture by Baudon, Bensmail, Przybyło, and Wozniak stating that if a graph can be decomposed into locally irregular graphs, then there exists such a decomposition with at most 3 parts. We show that this conjecture is true if the number 3 is replaced by 328, establishing the first constant upper bound for this problem.
Decomposition of Graphs

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