Distinguishability Revisited: Depth Dependent Bounds on Reconstruction Quality in Electrical Impedance Tomography:

Depth dependent bounds on reconstruction quality in electrical impedance tomography

The reconstruction problem in electrical impedance tomography is highly ill-posed, and it is often observed numerically that reconstructions have poor resolution far away from the measurement boundary but better resolution near the measurement boundary. The observation can be quantified by the concept of distinguishability of inclusions. This paper provides mathematically rigorous results supporting the intuition. Indeed, for a model problem lower and upper bounds on the distinguishability of an inclusion are derived in terms of the boundary data. These bounds depend explicitly on the distance of the inclusion to the boundary, i.e. the depth of the inclusion. The results are obtained for disk inclusions in a homogeneous background in the unit disk. The theoretical bounds are verified numerically using a novel, exact characterization of the forward map as a tridiagonal matrix.
Iterative Reconstruction Methods for Inverse Problems in Tomography with Hybrid Data

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Propagation of singularities for linearised hybrid data impedance tomography
For a general formulation of linearised hybrid inverse problems in impedance tomography, the qualitative properties of the solutions are analysed. Using an appropriate scalar pseudo-differential formulation, the problems are shown to permit propagating singularities under certain non-elliptic conditions, and the associated directions of propagation are precisely identified relative to the directions in which ellipticity is lost. The same result is found in the setting for the corresponding normal formulation of the scalar pseudo-differential equations. A numerical reconstruction procedure based of the least squares finite element method is derived, and a series of numerical experiments visualise exactly how the loss of ellipticity manifests itself as propagating singularities.
Hybrid data, LSFEM, Reconstruction algorithm, Impedance tomography, Coupled physics tomography, Ultrasound modulated EIT, Acousto-electric tomography
Segmentation-Driven Tomographic Reconstruction.

The tomographic reconstruction problem is concerned with creating a model of the interior of an object from some measured data, typically projections of the object. After reconstructing an object it is often desired to segment it, either automatically or manually. For computed tomography (CT), the classical reconstruction methods suffer from their inability to handle limited and/or corrupted data. Form any analysis tasks computationally demanding segmentation methods are used to automatically segment an object, after using a simple reconstruction method as a first step. In the literature, methods that completely combine reconstruction and segmentation have been suggested, but these are often non-convex and have very high computational demand. We propose to move the computational effort from the segmentation process to the reconstruction process, and instead design reconstruction methods such that the segmentation subsequently can be carried out by use of a simple segmentation method, for instance just a thresholding method. We tested the advantages of going from a two-stage reconstruction method to a one stage segmentation-driven reconstruction method for the phase contrast tomography reconstruction problem. The tests showed a clear improvement for realistic materials simulations and that the one-stage method was clearly more robust toward noise. The noise-robustness result could be a step toward making this method more applicable for lab-scale experiments. We have introduced a segmentation-driven reconstruction method which incorporates information about the main texture direction in an object. We proved that this method has mathematically desirable properties such as being convex and lower semicontinuous. We have also demonstrated the practical applicability of the method.

within image denoising, image deblurring and CT reconstruction. In order to use the proposed method we also proposed efficient and robust methods for estimating the main direction in either corrupted images or from limited and corrupted CT projection data. For directional object we also proposed two different reconstruction methods that separates the directional parts in the object from the non-directional parts. These method could for example be used for objects consisting of fibres and cracks. The results can be categorized as either completely combined reconstruction and segmentation of the object, or as highly supporting for the following segmentation process. Computed tomography is used within medical diagnosis, food science, materials science, production inspection, quality assessment, etc. Segmentation-driven reconstruction methods can help improve both manual and automated segmentation processes that are used to analyze an object after the scanning. The results in this thesis are both of theoretical interest within regularization theory and directly applicable for image denoising, image deblurring and surely within tomographic reconstruction.

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Prior Information in Inverse Boundary Problems
This thesis gives a threefold perspective on the inverse problem of inclusion detection in electrical impedance tomography: depth dependence, monotonicity-based reconstruction, and sparsity-based reconstruction.

The depth dependence is given in terms of explicit bounds on the datum norm, which shows the change in distinguishability of inclusions (support of an inhomogeneity) as they are placed closer towards the measurement boundary. This is done by determining eigenvalue bounds for differences of pseudodifferential operators on the boundary of the domain. Ultimately, the bounds serves as insight into how much noise that can be allowed in the datum before an inclusion cannot be detected.
The monotonicity method is a direct reconstruction method that utilizes a monotonicity property of the forward problem in order to characterize the inclusions. Here we rigorously prove that the method can be regularized against noise with a uniform regularization parameter, and that the method can be generalized to discrete electrode models. We give examples in 2D and 3D with noisy simulated data as well as real measurements, and give a comparison of reconstructions based on a non-linear and a linear formulation of the method.

Sparsity-based reconstruction is an iterative method, that through an optimization problem with a sparsity prior, approximates the inhomogeneities. Here we make use of prior information, that can cheaply be obtained from the monotonicity method, to improve both the contrast and resolution of the reconstruction. Numerical examples are given in both 2D and 3D for partial data using noisy simulated data as well as real measurements.

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Authors: Garde, H. (Intern), Knudsen, K. (Intern)
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Publication: Research › Ph.D. thesis – Annual report year: 2016

Sparsity prior for electrical impedance tomography with partial data
This paper focuses on prior information for improved sparsity reconstruction in electrical impedance tomography with partial data, i.e. Cauchy data measured on subsets of the boundary. Sparsity is enforced using an (Formula presented.) norm of the basis coefficients as the penalty term in a Tikhonov functional, and prior information is incorporated by applying a spatially distributed regularization parameter. The resulting optimization problem allows great flexibility with respect to the choice of measurement subsets of the boundary and incorporation of prior knowledge. In fact, the measurement subsets can be chosen completely arbitrary. The problem is solved using a generalized conditional gradient method applying soft thresholding. Numerical examples with noisy simulated data show that the addition of prior information in the proposed algorithm gives vastly improved reconstructions, even for the partial data problem. Moreover, numerical examples show that a reliable reconstruction for the partial data problem can only be found close to the measurement subsets. The method is in addition compared to a total variation approach.

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Organisations: Department of Applied Mathematics and Computer Science , Scientific Computing
Authors: Garde, H. (Intern), Knudsen, K. (Intern)
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Web of Science (2018): Indexed yes
BFI (2017): BFI-level 1
Web of Science (2017): Indexed Yes
BFI (2016): BFI-level 1
3D reconstruction for partial data electrical impedance tomography using a sparsity prior

In electrical impedance tomography, the electrical conductivity inside a physical body is computed from electro-static boundary measurements. The focus of this paper is to extend recent results for the 2D problem to 3D: prior information about the sparsity and spatial distribution of the conductivity is used to improve reconstructions for the partial data problem with Cauchy data measured only on a subset of the boundary. A sparsity prior is enforced using the $\ell_1$ norm in the penalty term of a Tikhonov functional, and spatial prior information is incorporated by applying a spatially distributed regularization parameter. The optimization problem is solved numerically using a generalized conditional gradient method with soft thresholding. Numerical examples show the effectiveness of the suggested method even for the partial data problem with measurements affected by noise.

General information
State: Published
Organisations: Department of Applied Mathematics and Computer Science, Scientific Computing
Authors: Garde, H. (Intern), Knudsen, K. (Intern)
High-resolution imaging methods in array signal processing

The purpose of this study is to develop methods in array signal processing which achieve accurate signal reconstruction from limited observations resulting in high-resolution imaging. The focus is on underwater acoustic applications and sonar signal processing both in active (transmit and receive) and passive (only receive) mode. The study addresses the limitations of existing methods and shows that, in many cases, the proposed methods overcome these limitations and outperform traditional methods for acoustic imaging.

The project comprises two parts; The first part deals with computational methods in active sonar signal processing for detection and imaging of submerged oil contamination in sea water from a deep-water oil leak. The submerged oil field is modeled as a uid medium exhibiting spatial perturbations in the acoustic parameters from their mean ambient values which cause weak scattering of the incident acoustic energy. A highfrequency active sonar is selected to insonify the medium and receive the backscattered waves. High-frequency acoustic methods can both overcome the optical opacity of water (unlike methods based on electromagnetic waves) and resolve the small-scale structure of the submerged oil field (unlike low-frequency acoustic methods). The study shows that high-frequency acoustic methods are suitable not only for large-scale localization of the oil contamination in the water column but also for statistical characterization of the submerged oil field through inference of the spatial covariance of its acoustic parameters.

The second part of the project investigates methods that exploit sparsity in order to achieve super-resolution in sound source localization with passive sonars. Sound source localization with sensor arrays involves the estimation of the direction-of-arrival (DOA) of the associated wavefronts from a limited number of observations. Usually, there are only a few sources generating the acoustic wavefield such that DOA estimation is essentially a sparse signal reconstruction problem. Conventional methods for DOA estimation (i.e., beamforming) suffer from resolution limitations related to the physical size and the geometry of the array. DOA estimation methods that are developed up-to-date in order to overcome the resolution limitations of conventional methods involve the estimation or the eigendecomposition of the data cross-spectral matrix. The cross-spectral methods require many snapshots (i.e., observation windows of the recorded wavefield) hence are suitable only for stationary incoherent sources. In this study, the DOA estimation problem is formulated both for single and multiple snapshots in the compressive sensing framework (CS), which achieves sparsity, thus improved resolution, and can be solved efficiently with convex optimization. It is shown that CS has superior performance compared to traditional DOA estimation methods especially under challenging scenarios such as coherent arrivals, single-snapshot data and random array configurations. The high-resolution performance and the robustness of CS in DOA estimation are validated with experimental array data from ocean acoustic measurements.
Reconstruction Methods for Inverse Problems with Partial Data

This thesis presents a theoretical and numerical analysis of a general mathematical formulation of hybrid inverse problems in impedance tomography. This includes problems from several existing hybrid imaging modalities such as Current Density Impedance Imaging, Magnetic Resonance Electrical Impedance Tomography, and Ultrasound Modulated Electrical Impedance Tomography. After giving an introduction to hybrid inverse problems in impedance tomography and the mathematical tools that facilitate the related analysis, we explain in detail the stability properties associated with the classification of a linearised hybrid inverse problem. This is done using pseudo-differential calculus and theory for overdetermined boundary value problem. Using microlocal analysis we then present novel results on the propagation of singularities, which give a precise description of the distinct features of solutions in the case of a non-elliptic problem. To conduct a numerical analysis, we develop four iterative reconstruction methods using the Picard and Newton iterative schemes, and the unified approach to the reconstruction problem encompasses several algorithms suggested in the literature. The algorithms are implemented numerically in two dimensions and the properties of the algorithms and their implementations are investigated theoretically. Novel numerical results are presented for both the full and partial data problem, and they show similarities and differences between the proposed algorithms, which are closely linked to the results of the theoretical analysis. The findings in this thesis justify that the choice of algorithm should be based on a theoretical analysis of the underlying inverse problem.

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Iterative Reconstruction Methods for Hybrid Inverse Problems in Impedance Tomography

For a general formulation of hybrid inverse problems in impedance tomography the Picard and Newton iterative schemes are adapted and four iterative reconstruction algorithms are developed. The general problem formulation includes several existing hybrid imaging modalities such as current density impedance imaging, magnetic resonance electrical impedance tomography, and ultrasound modulated electrical impedance tomography, and the unified approach to the reconstruction problem encompasses several algorithms suggested in the literature. The four proposed algorithms are implemented numerically in two dimensions and the properties of the algorithms and the implementations are investigated, both theoretically and on simulated data obtained from a numerical phantom. The numerical results show similarities and differences between the proposed algorithms, and they justify that the choice of algorithm should be based on a theoretical analysis of the underlying inverse problem.

General information
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Organisations: Department of Applied Mathematics and Computer Science, Scientific Computing
Authors: Hoffmann, K. (Intern), Knudsen, K. (Intern)
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Numerical nonlinear complex geometrical optics algorithm for the 3D Calderón problem

The Calderon problem is the mathematical formulation of the inverse problem in Electrical Impedance Tomography and asks for the uniqueness and reconstruction of an electrical conductivity distribution in a bounded domain from the knowledge of the Dirichlet-to-Neumann map associated to the generalized Laplace equation. The 3D problem was solved in theory in late 1980s using complex geometrical optics solutions and a scattering transform. Several approximations to the reconstruction method have been suggested and implemented numerically in the literature, but here, for the first time, a complete computer implementation of the full nonlinear algorithm is given. First a boundary integral equation is solved by a Nystrom method for the traces of the complex geometrical optics solutions, second the scattering transform is computed and inverted using fast Fourier transform, and finally a boundary value problem is solved for the conductivity distribution. To test the performance of the algorithm highly accurate data is required, and to this end a boundary element method is developed and implemented for the forward problem. The numerical reconstruction algorithm is tested on simulated data and compared to the simpler approximations. In addition, convergence of the numerical solution towards the exact solution of the boundary integral equation is proved.

General information
State: Published
Organisations: Department of Applied Mathematics and Computer Science, Scientific Computing, University of Genoa
Authors: Delbary, F. (Intern), Knudsen, K. (Intern)
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Size-based predictions of food web patterns

We employ size-based theoretical arguments to derive simple analytic predictions of ecological patterns and properties of natural communities: size-spectrum exponent, maximum trophic level, and susceptibility to invasive species. The predictions are brought about by assuming that an infinite number of species are continuously distributed on a size-trait axis. It is, however, an open question whether such predictions are valid for a food web with a finite number of species embedded in a network structure. We address this question by comparing the size-based predictions to results from dynamic food web simulations with varying species richness. To this end, we develop a new size- and trait-based food web model that can be simplified into an analytically solvable size-based model. We confirm existing solutions for the size distribution and derive novel predictions for maximum trophic level and invasion resistance. Our results show that the predicted size-spectrum exponent is borne out in the simulated food webs even with few species, albeit with a systematic bias. The predicted maximum trophic level turns out to be an upper limit since simulated food webs may have a lower number of trophic levels, especially for low species richness, due to structural constraints. The size-based model possesses an evolutionary stable state and is therefore un-invadable. In contrast, the food web simulations show that all communities, irrespective of number of species, are equally open to invasions. We use these results to discuss the validity of size-based predictions in the light of the structural constraints imposed by food webs.

General information

State: Published
Organisations: Department of Applied Mathematics and Computer Science, National Institute of Aquatic Resources, Centre for Ocean Life, Scientific Computing, Section for Marine Ecology and Oceanography
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Inclusion estimation from a single electrostatic boundary measurement: Paper

We present a numerical method for the detection and estimation of perfectly conducting inclusions in conducting homogeneous host media in . The estimation is based on the evaluation of an indicator function that depends on a single pair of Cauchy data (electric potential and current) given at the boundary of the medium. The indicator function is derived using Green’s third identity with the fundamental solution for the Dirichlet Laplacian on the unit disc. Using a truncated Taylor expansion, the indicator function is expressed in terms of an integral over a perturbed inclusion boundary, resulting in a natural physical interpretation. The method is implemented numerically, tested on different example problems and compared to a decomposition approach based on the method of fundamental solutions. The method shows promising results and seems robust to noisy, low sampling-frequency data.

General information
State: Published
Organisations: Department of Applied Mathematics and Computer Science, Scientific Computing
Authors: Karamehmedovic, M. (Intern), Knudsen, K. (Intern)
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Pages: 025005
Publication date: 2013
Main Research Area: Technical/natural sciences
Trait diversity promotes stability of community dynamics

The theoretical exploration of how diversity influences stability has traditionally been approached by species-centric methods. Here we offer an alternative approach to the diversity–stability problem by examining the stability and dynamics of size and trait distributions of individuals. The analysis is performed by comparing the properties of two size spectrum models. The first model considers all individuals as belonging to the same “average” species, i.e., without a description of
diversity. The second model introduces diversity by further considering individuals by a trait, here asymptotic body size. The dynamic properties of the models are described by a stability analysis of equilibrium solutions and by the non-equilibrium dynamics. We find that the introduction of trait diversity expands the set of parameters for which the equilibrium is stable and, if the community is unstable, makes the oscillations smaller, slower, and more regular. The stabilizing mechanism is the variation in growth rate between individuals with the same body size but different trait values.

General information
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Organisations: Applied functional analysis, National Institute of Aquatic Resources, Department of Mathematics, Section for Population Ecology and Genetics
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Web of Science (2016): Indexed yes
BFI (2015): BFI-level 1
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BFI (2014): BFI-level 1
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Web of Science (2014): Indexed yes
BFI (2013): BFI-level 1
Scopus rating (2013): SJR 1.456 SNIP 0.914 CiteScore 2.2
ISI indexed (2013): ISI indexed yes
Web of Science (2013): Indexed yes
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ISI indexed (2012): ISI indexed yes
BFI (2011): BFI-level 1
Scopus rating (2011): SJR 1.341 SNIP 0.696 CiteScore 1.74
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Viscoelastic Modelling of Road Deflections for use with the Traffic Speed Deflectometer
This Ph.D. study is at its core about how asphalt and road structures responds to dynamic loads. Existing models for the deflections under a moving load using beam equations are revisited and it is concluded they leave room for improvement for the particular setup and problem at hand. Then a different approach is set up to model viscoelastic deflections starting from the physically based framework of continuum mechanics by using Finite Element Methods (FEM) combined with the Laplace transform. It is shown that this approach coincides with a more standard time-stepping FEM setup in the case of a generalized Maxwell model.
Validations by comparison to ViscoRoute simulations are also made. This justifies the use of the Laplace FEM for generating simulated data using a Huet-Sayegh model for the visco-elastic behaviour of asphalt. These simulated data, along with measured data, are then used to suggest an approach for a computationally simpler synthetic model capturing essential behaviour of deflection basins under a moving wheel.

Additionally the setup allows for simulated comparisons of the cases of loadings emulating the use of a Falling Weight Deflectometer with loadings emulating a moving wheel as in the case of using a Traffic Speed Deflectometer. The flexibility of the method also allows for looking into cases excluded by imposing simplifying assumptions such as the structure imagined to be an infinite halfspace.

**General information**

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Organisations: Department of Applied Mathematics and Computer Science, Dynamical Systems, Scientific Computing
Authors: Pedersen, L. (Intern), Hjorth, P. G. (Intern), Knudsen, K. (Intern)
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**Electrical Impedance Tomography: 3D Reconstructions using Scattering Transforms**

In three dimensions the Calderon problem was addressed and solved in theory in the 1980s. The main ingredients in the solution of the problem are complex geometrical optics solutions to the conductivity equation and a (non-physical) scattering transform. The resulting reconstruction algorithm is in principle direct and addresses the full non-linear problem immediately. In this paper a new simplication of the algorithm is suggested. The method is based on solving a boundary integral equation for the complex geometrical optics solutions, and the method is implemented numerically using a Nystrom method. Convergence estimates are obtained using hyperinterpolation operators. We compare the method numerically to two other approximations by evaluation on two numerical examples. In addition a moment method for the numerical solution of the forward problem is given.

**General information**

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Organisations: Department of Informatics and Mathematical Modeling, Scientific Computing, Applied functional analysis, Department of Mathematics
Authors: Delbary, F. (Intern), Hansen, P. C. (Intern), Knudsen, K. (Intern)
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Web of Science (2017): Indexed Yes
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Scopus rating (2016): SJR 0.815 SNIP 0.848 CiteScore 0.83
BFI (2015): BFI-level 1
Noise mapping inside a car cabin
The mapping of noise is of considerable interest in the car industry where a good noise mapping can make it much easier to identify the sources that generate the noise and eventually reduce the individual contributions to the noise. The methods used for this purpose include delay-and-sum beamforming and spherical harmonics beamforming. These methods have a poor spatial resolution at low frequencies, and since much noise generated in cars is dominated by low frequencies the methods are not optimal. In the present paper the mapping is done by solving an inverse problem with a transfer matrix between the volume velocities of the sources and the measured sound pressures at the microphone array. This is an illposed problem and therefore regularisation have to be applied when the transfer matrix is inverted in order to give good results.

General information
State: Published
Organisations: Department of Mathematics, Applied functional analysis, Department of Electrical Engineering, Acoustic Technology, Technical University of Denmark, Brüel & Kjær A/S
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Number of pages: 6
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Source-ID: u::6615
Publication: Research - peer-review › Paper – Annual report year: 2012

Numerical Reconstruction of Perfectly Conducting Inclusions from One Electrostatic Boundary Measurement
Electrical Impedance Tomography (EIT) aims to reconstruct inhomogeneities or inclusions in the interior of a medium based on current and voltage measurements at the boundary of the medium. In many practical applications of EIT, the efficiency of the employed numerical inversion scheme is an essential parameter. The inversion methods typically evaluate
an indicator function in order to estimate whether or not a given point is in the interior of the sought inclusion. The so-called sampling methods do
not assume any a priori knowledge about the boundary condition valid at the inclusion boundary, but the evaluation of their indicator functions can be numerically expensive. On the other hand, decomposition methods express the solution of the Laplace equation in the medium in terms of layer potentials and estimate the inclusion using boundary value error minimisation. However, matching the sources in the potentials with the measured current or voltage at the medium boundary can be numerically costly.

We describe a novel method for the reconstruction of perfectly electrically conducting inclusions in arbitrary homogeneous, simply connected media of finite conductivity and with sufficiently smooth boundary. Similarly to the decomposition methods, our approach is based on a boundary layer representation of a solution of the Laplace equation in the medium, and it uses a priori knowledge of the boundary condition satisfied at the interface between the inclusion and the medium. However, both of these developments occur at the analytic stage only, and the actual numerical computation involves neither forward-model sources nor boundary-error minimisation. The method requires inclusions to be placed relatively close to the boundary of the medium, and it is well-suited for detection of small inclusions and for detection and partial shape estimation of large inclusions. We give a mathematical justification for the indicator function used in the inversion method. Also, we illustrate the performance of the method using several numerical examples involving different medium geometries, as well as single and multiple inclusions of different shapes and positions within the medium. Finally, we compare the efficiency and accuracy of the method to a decomposition scheme based on the Method of Auxiliary Sources.

A Direct Numerical Reconstruction Algorithm for the 3D Calderón Problem
In three dimensions Calderón's problem was addressed and solved in theory in the 1980s in a series of papers, but only recently the numerical implementation of the algorithm was initiated. The main ingredients in the solution of the problem are complex geometrical optics solutions to the conductivity equation and a (non-physical) scattering transform. The resulting reconstruction algorithm is in principle direct and addresses the full non-linear problem immediately. In this paper we will outline the theoretical reconstruction method and describe how the method can be implemented numerically. We will give three different implementations, and compare their performance on a numerical phantom.
Direct numerical reconstruction of conductivities in three dimensions using scattering transforms

A direct three-dimensional EIT reconstruction algorithm based on complex geometrical optics solutions and a nonlinear scattering transform is presented and implemented for spherically symmetric conductivity distributions. The scattering transform is computed both with a Born approximation and from the forward problem for purposes of comparison. Reconstructions are computed for several test problems. A connection to Calderón's linear reconstruction algorithm is established, and reconstructions using both methods are compared.

General information
State: Published
Organisations: Department of Mathematics, Applied functional analysis, Colorado State University
Authors: Bikowski, J. (Intern), Knudsen, K. (Intern), Mueller, J. L. (Ekstern)
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Main Research Area: Technical/natural sciences

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Web of Science (2018): Indexed yes
The born approximation and Calderón's method for reconstruction of conductivities in 3-D
Two algorithms for the direct reconstruction of conductivities in a bounded domain in \([\mathbb{R}^3]\) from surface measurements of the solutions to the conductivity equation are presented. The algorithms are based on complex geometrical optics solutions and a nonlinear scattering transform. We test the algorithms on three numerically simulated examples, including an example with a complex coefficient. The spatial resolution and amplitude of the examples are well-reconstructed.

General information
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Organisations: Department of Mathematics, Applied functional analysis, Colorado State University
Authors: Knudsen, K. (Intern), Mueller, J. L. (Ekstern)
Pages: 844-853
Numerical Approximation of Boundary Control for the Wave Equation - with Application to an Inverse Problem

We consider a control problem for the wave equation: Given the initial state, find a specific boundary condition, called a control, that steers the system to a desired final state. The Hilbert uniqueness method (HUM) is a mathematical method for the solution of such control problems. It builds on the duality between the control system and its adjoint system, and these systems are connected via a so-called controllability operator. In this project, we are concerned with the numerical approximation of HUM control for the one-dimensional wave equation. We study two semi-discretizations of the wave equation: a linear finite element method (L-FEM) and a discontinuous Galerkin-FEM (DG-FEM). The controllability operator is discretized with both L-FEM and DG-FEM to obtain a HUM matrix. We show that formulating HUM in a sine basis is beneficial for several reasons: (i) separation of low and high frequency waves, (ii) close connection to the dispersive relation, (iii) simple and effective filtering. The dispersive behavior of a discretization is very important for its ability to solve control problems. We demonstrate that the group velocity is determining for a scheme’s success in relation to HUM. The vanishing group velocity for high wavenumbers results in a dramatic decay of the corresponding eigenvalues of the HUM matrix and thereby also in a huge condition number. We show that, provided sufficient filtering, the phase velocity decides the accuracy of the computed controls. DG-FEM shows very suitable for the treatment of control problems. The good dispersive behavior is an important virtue and a decisive factor in the success over L-FEM. Increasing the order of DG-FEM even give results of spectral accuracy. The field of control is closely related to other fields of mathematics among these are inverse problems. As an example, we employ a HUM solution to an inverse source problem for the wave equation: Given boundary measurements for a wave problem with a separable source, find the spatial part of the source term. The reconstruction formula depends on a set of HUM eigenfunction controls; we suggest a discretization and show its convergence. We compare results obtained by L-FEM controls and DG-FEM controls. The reconstruction formula is seen to be quite sensitive to control inaccuracies which indeed favors DG-FEM over L-FEM.

REGULARIZED D-BAR METHOD FOR THE INVERSE CONDUCTIVITY PROBLEM

A strategy for regularizing the inversion procedure for the two-dimensional D-bar reconstruction algorithm based on the global uniqueness proof of Nachman [Ann. Math. 143 (1996)] for the ill-posed inverse conductivity problem is presented. The strategy utilizes truncation of the boundary integral equation and the scattering transform. It is shown that this leads to a bound on the error in the scattering transform and a stable reconstruction of the conductivity; an explicit rate of convergence in appropriate Banach spaces is derived as well. Numerical results are also included, demonstrating the convergence of the reconstructed conductivity to the true conductivity as the noise level tends to zero. The results provide a link between two traditions of inverse problems research: theory of regularization and inversion methods based on...
complex geometrical optics. Also, the procedure is a novel regularized imaging method for electrical impedance tomography.

Reconstructions of Piecewise Constant Conductivities by the D-bar Method for Electrical Impedance Tomography.

Reconstructions of Piecewise Constant Conductivities by the D-bar Method for Electrical Impedance Tomography.

Reconstructions of Piecewise Constant Conductivities by the D-bar Method for Electrical Impedance Tomography.
Projects:

**Quantity of Interest Tomography**

Department of Applied Mathematics and Computer Science  
Period: 01/01/2017 → 31/12/2019  
Number of participants: 4  
Phd Student:  
Jensen, Bjørn Christian Skov (Intern)  
Supervisor:  
Adesokan, Bolaji James (Intern)  
Andersen, Martin Skovgaard (Intern)  
Main Supervisor:  
Knudsen, Kim (Intern)

**Financing sources**  
Source: Internal funding (public)  
Name of research programme: Institut stipendie (DTU)  
Project: PhD

**Mathematical Analysis and Computations for Multiphysics Tomography**

Department of Applied Mathematics and Computer Science  
Period: 15/12/2015 → 16/05/2019  
Number of participants: 4  
Phd Student:  
Kirkeby, Adrian (Intern)  
Supervisor:  
Evgrafov, Anton (Intern)  
Karamehmedovic, Mirza (Intern)  
Main Supervisor:  
Knudsen, Kim (Intern)

**Financing sources**  
Source: Internal funding (public)  
Name of research programme: Institut stipendie (DTU)  
Project: PhD

**Numerical Inversion Methods for Impedance tomography with hybrid data**

Department of Applied Mathematics and Computer Science  
Period: 01/01/2015 → 31/12/2017  
Number of participants: 5  
Phd Student:  
Sherina, Ekaterina (Intern)  
Main Supervisor:
PhD Project: Segmentation-Driven Tomographic Reconstruction

Computed tomography (CT) is a non-invasive technique for analyzing the interior of objects. The mathematical method of calculating the interior of an object is called reconstruction. A great variety of different reconstruction techniques exist. For this project the goal is to incorporate different forms of prior information into the reconstruction process to achieve results with desired features for a subsequent segmentation. The CT problem is an ill-posed problem, which is a motivation for incorporation prior information, in order to regularize and stabilize the reconstructions. Prior information is based on what we perceive as expected and typical behavior for specific problems, for example an often-used prior for CT reconstructions is piecewise constancy of the solutions, which is utilized by for example Total Variation regularization. Incorporation of prior information in reconstructions is also a part of the overall theme for the ERC project “High-Definition Tomography”, which this project is a part of.

CT is typically used for analyzing biological objects, for medical imaging purposes, though in the research field of material science this has also become a highly used technique. For materials science a typical CT-investigation pipeline consist of four major stages: scanning, reconstruction, segmentation and analysis. Often the reconstruction is carried out by a simple filtered back projection method, whereas the segmentation stage consists of more advanced and computationally expensive methods.

In my project we aim to move the computational effort from the segmentation stage to the reconstruction stage. The reconstruction methods that primarily investigate are related to the variational methods. Prior information about the object we are scanning is used to regularize the reconstruction in order aid the following segmentation stage. Some regularization keywords that I have been working with are: Total Variation, Directional Total Variation, Total Generalized Variation, Mumford-Shah and Eulers Elastica.

Department of Applied Mathematics and Computer Science

Scientific Computing

Period: 01/09/2014 → 31/08/2017
Number of participants: 3
Number of related Ph.D. students: 1
Project participant:
Kongskov, Rasmus Dalgas (Intern)
Dong, Yiqiu (Intern)
Knudsen, Kim (Intern)

Project
Segmentation-Driven Tomographic Reconstruction

Department of Applied Mathematics and Computer Science  
Period: 01/09/2014 → 15/11/2017  
Number of participants: 6  
Phd Student: Kongskov, Rasmus Dalgas (Intern)  
Supervisor: Knudsen, Kim (Intern)  
Main Supervisor: Dong, Yiqiu (Intern)  
Examiner: Andersen, Martin Skovgaard (Intern)  
Guo, Weihong (Ekstern)  
Pock, Thomas Georg (Ekstern)

Financing sources  
Source: Internal funding (public)  
Name of research programme: Anden EU-finansiering

Relations  
Publications: Segmentation-Driven Tomographic Reconstruction. Project: PhD

Gabor Frames, their Duals and Applications in Engineering

Department of Applied Mathematics and Computer Science  
Period: 01/10/2013 → 23/11/2016  
Number of participants: 6  
Phd Student: Jakobsen, Mads Sielemann (Intern)  
Supervisor: Lemvig, Jakob (Intern)  
Main Supervisor: Christensen, Ole (Intern)  
Examiner: Knudsen, Kim (Intern)  
Feichtinger, Hans (Intern)  
Hernández, Eugenio (Ekstern)

Financing sources  
Source: Internal funding (public)  
Name of research programme: Institut stipendie (DTU)

Relations  
Publications: Gabor frames on locally compact abelian groups and related topics  
Project: PhD

Prior Information in Inverse Boundary problems

Department of Applied Mathematics and Computer Science  
Period: 01/03/2013 → 25/05/2016  
Number of participants: 5  
Phd Student: Garde, Henrik (Intern)  
Main Supervisor: Knudsen, Kim (Intern)  
Examiner:
Reconstruction methods for inverse problems with partial data

Department of Applied Mathematics and Computer Science
Number of participants: 5
Phd Student:
Hoffmann, Kristoffer (Intern)
Main Supervisor:
Knudsen, Kim (Intern)
Examiner:
Markvorsen, Steen (Intern)
Uhlmann, Gunther (Ekstern)
von Harrach, Bastian (Ekstern)

Computational methods for detection and imaging of oil in sea water

Department of Applied Mathematics and Computer Science
Period: 01/07/2011 → 13/08/2015
Number of participants: 7
Phd Student:
Xenaki, Angeliki (Intern)
Supervisor:
Brunkog, Jonas (Intern)
Mosegaard, Klaus (Intern)
Main Supervisor:
Knudsen, Kim (Intern)
Examiner:
Agerkvist, Finn T. (Intern)
Edelmann, Geoffrey F. (Ekstern)
Nordborg, Anders (Ekstern)

Mathematical Road Models for the Traffic Speed Deflectometer

Department of Informatics and Mathematical Modeling
Period: 01/09/2009 → 22/03/2013
Number of participants: 6
Phd Student:
Pedersen, Louis (Intern)
Supervisor:
Knudsen, Kim (Intern)
Main Supervisor:
Mathematical modelling and analysis of marine ecological systems with stage structures and size spectra

Department of Mathematics
Period: 01/09/2008 → 23/02/2012
Number of participants: 7
Phd Student:
Zhang, Lai (Intern)
Supervisor:
Andersen, Ken Haste (Intern)
Thygesen, Uffe Høgsbro (Intern)
Main Supervisor:
Knudsen, Kim (Intern)
Examiner:
Starke, Jens (Intern)
25_NN_Studenter/Øvrige medarb. (Ekstern)
Delius, Gustav W. (Ekstern)

Financing sources
Source: Internal funding (public)
Name of research programme: ErhvervsPhD-ordningen VTU
Project: PhD

Multiphase Flow Evaluation

Department of Mechanical Engineering
Period: 01/11/2006 → 30/09/2010
Number of participants: 6
Phd Student:
Kjærsgaard-Rasmussen, Jimmy (Intern)
Supervisor:
Hallundbæk, Jørgen (Ekstern)
Main Supervisor:
Meyer, Knud Erik (Intern)
Examiner:
Knudsen, Kim (Intern)
Meyer, Stefan (Intern)
Vauhkonen, Marko (Ekstern)

Financing sources
Source: Internal funding (public)
Name of research programme: Institut stipendie (DTU) Samf.
Project: PhD

Numerisk approksimation af rand-kontrol problemer

Department of Mathematics
Number of participants: 7
Phd Student:
Mariegaard, Jesper Sandvig (Intern)
Supervisor:
Hansen, Per Christian (Intern)
Pedersen, Michael (Intern)
Main Supervisor:
Knudsen, Kim (Intern)
Examiner:
Sørensen, Mads Peter (Intern)
Hesthaven, Jan (Intern)
Hugger, Jens (Ekstern)

Financing sources
Source: Internal funding (public)
Name of research programme: DTU-lønnet stipendie
Project: PhD

Activities:

Hybrid data tomography
Period: 22 Jan 2018 → 26 Jan 2018
Tommi Olavi Brander (Organizer)
Kim Knudsen (Organizer)
Bjørn Christian Skov Jensen (Organizer)
Ekaterina Sherina (Organizer)
Department of Applied Mathematics and Computer Science
Scientific Computing

Related event

Hybrid data tomography
22/01/2018 → 26/01/2018
Kongens Lyngby, Denmark
Activity: Attending an event › Participating in or organising workshops, courses, seminars etc.