A Numerical Framework for Sobolev Metrics on the Space of Curves
Statistical shape analysis can be done in a Riemannian framework by endowing the set of shapes with a Riemannian metric. Sobolev metrics of order two and higher on shape spaces of parametrized or unparametrized curves have several desirable properties not present in lower order metrics, but their discretization is still largely missing. In this paper, we present algorithms to numerically solve the geodesic initial and boundary value problems for these metrics. The combination of these algorithms enables one to compute Karcher means in a Riemannian gradient-based optimization scheme and perform principal component analysis and clustering. Our framework is sufficiently general to be applicable to a wide class of metrics. We demonstrate the effectiveness of our approach by analyzing a collection of shapes representing HeLa cell nuclei.
Optimization on Spaces of Curves

This thesis is concerned with computational and theoretical aspects of Riemannian metrics on spaces of regular curves, and their applications. It was recently proved that second order constant coefficient Sobolev metrics on curves are geodesically complete. We extend this result to the case of Sobolev metrics with coefficient functions depending on the length of the curve. We show how to apply this result to analyse a wide range of metrics on the submanifold of unit and constant speed curves.

We present a numerical discretization of second order Sobolev metrics on the space of regular curves in $\mathbb{R}^d$, and methods to solve the initial and boundary value problem for geodesics allowing us to compute the Karcher mean and principal components analysis of data of curves. We apply the methods to study shape variation in synthetic data in the Kimia shape database, in HeLa cell nuclei and cycles of cardiac deformations.

Finally we investigate a new application of Riemannian shape analysis in shape optimization. We setup a simple elliptic model problem, and describe how to apply shape calculus to obtain directional derivatives in the manifold of planar curves. We present an implementation based on parametrization of immersions by B-splines, which ties in naturally with Isogeometric Analysis to solve the PDE. We give numerical examples of solutions, and compare the Riemannian optimization algorithms with different choices of metrics to a naive unregularized discretize-first approach.
Curve Matching with Applications in Medical Imaging
In the recent years, Riemannian shape analysis of curves and surfaces has found several applications in medical image analysis. In this paper we present a numerical discretization of second order Sobolev metrics on the space of regular curves in Euclidean space. This class of metrics has several desirable mathematical properties. We propose numerical solutions for the initial and boundary value problems of finding geodesics. These two methods are combined in a Riemannian gradient-based optimization scheme to compute the Karcher mean. We apply this to a study of the shape variation in HeLa cell nuclei and cycles of cardiac deformations, by computing means and principal modes of variations.

General information
State: Published
Organisations: Department of Applied Mathematics and Computer Science, Mathematics, Technische Universität Wien, Brunel University, ETH Zurich
Authors: Bauer, M. (Ekstern), Bruveris, M. (Ekstern), Harms, P. (Ekstern), Møller-Andersen, J. (Intern)
Pages: 83-94
Publication date: 2015

Host publication information
Title of host publication: Proceedings of the 5th MICCAI Workshop on Mathematical Foundations of Computational Anatomy (MFCA 2015)
Main Research Area: Technical/natural sciences
Workshop: 5th MICCAI Workshop on Mathematical Foundations of Computational Anatomy (MFCA 2015), Munich, Germany, 09/10/2015

Curve matching, Sobolev metrics, Riemannian shape analysis, Discrete geodesics, Minimizing geodesics

Electronic versions:
CurveMatchingMiccai2015.pdf

Links:

Bibliographical note
The proceedings of the workshop are available as a collection of open archive papers.

Source: PublicationPreSubmission
Source-ID: 118716831
Publication: Research - peer-review › Article in proceedings – Annual report year: 2015

Perturbative Semiclassical Trace Formulae for Harmonic Oscillators
In this article we extend previous semiclassical studies by including more general perturbative potentials of the harmonic oscillator in arbitrary spatial dimensions. Our starting point is a radial harmonic potential with an arbitrary even monomial perturbation, which we use to study the resulting U(D) to O(D) symmetry breaking. We derive the gross structure of the semiclassical spectrum from periodic orbit theory, in the form of a perturbative (ℏ → 0) trace formula. We then show how to apply the results to even-order polynomial potentials, possibly including mean-field terms. We have drawn the conclusion that the gross structure of the quantum spectrum is determined from only classical circular and diameter orbits for this class of systems.

General information
State: Published
Organisations: Department of Applied Mathematics and Computer Science, Mathematics
Authors: Møller-Andersen, J. (Intern), Ögren, M. (Intern)
Number of pages: 24
Pages: 359-382
Publication date: 2015
Main Research Area: Technical/natural sciences

Publication information
Journal: Reports on Mathematical Physics
Volume: 75
Issue number: 3
ISSN (Print): 0034-4877
Ratings:
BFI (2018): BFI-level 1
BFI (2017): BFI-level 1
Web of Science (2017): Indexed Yes
Second order elastic metrics on the shape space of curves

Second order Sobolev metrics on the space of regular unparametrized planar curves have several desirable completeness properties not present in lower order metrics, but numerics are still largely missing. In this paper, we present algorithms to numerically solve the initial and boundary value problems for geodesics. The combination of these algorithms allows to compute Karcher means in a Riemannian gradient-based optimization scheme. Our framework has the advantage that the constants determining the weights of the zero, first, and second order terms of the metric can be chosen freely. Moreover, due to its generality, it could be applied to more general spaces of mapping. We demonstrate the effectiveness of our approach by analyzing a collection of shapes representing physical objects.

General information
State: Published
Organisations: Department of Applied Mathematics and Computer Science, Mathematics, Technische Universität Wien, Brunel University, ETH Zurich
Authors: Bauer, M. (Ekstern), Bruveris, M. (Ekstern), Harms, P. (Ekstern), Møller-Andersen, J. (Intern)
Pages: 1-11
Publication date: 2015

Host publication information
Title of host publication: Proceedings of the 1st International Workshop on DIFFerential Geometry in Computer Vision for Analysis of Shapes, Images and Trajectories (DIFF-CV) 2015
Publisher: BMVA Press
Editors: Drira, H., Kurtek, S., Turaga, P.
Projects:

Optimization on Manifolds - with applications to shape optimization.

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Number of participants: 7
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Financing sources
Source: Internal funding (public)
Name of research programme: Institut stipendie (DTU)

Relations
Publications:
Optimization on Spaces of Curves
Project: PhD