Canards in stiction: on solutions of a friction oscillator by regularization
We study the solutions of a friction oscillator subject to stiction. This discontinuous model is non-Filippov, and the concept of Filippov solution cannot be used. Furthermore some Carathéodory solutions are unphysical. Therefore we introduce the concept of stiction solutions: these are the Carathéodory solutions that are physically relevant, i.e. the ones that follow the stiction law. However, we find that some of the stiction solutions are forward non-unique in subregions of the slip onset. We call these solutions singular, in contrast to the regular stiction solutions that are forward unique. In order to further the understanding of the non-unique dynamics, we introduce a regularization of the model. This gives a singularly perturbed problem that captures the main features of the original discontinuous problem. We identify a repelling slow manifold that separates the forward slipping to forward sticking solutions, leading to a high sensitivity to the initial conditions. On this slow manifold we find canard trajectories, that have the physical interpretation of delaying the slip onset. We show with numerics that the regularized problem has a family of periodic orbits interacting with the canards. We observe that this family has a saddle stability and that it connects, in the rigid body limit, the two regular, slip-stick branches of the discontinuous problem, that were otherwise disconnected.
This thesis is concerned with the application of geometric singular perturbation theory to mechanical systems with friction. The mathematical background on geometric singular perturbation theory, on the blow-up method, on non-smooth dynamical systems and on regularization is presented. Thereafter, two mechanical problems with two different formulations of the friction force are introduced and analysed. The first mechanical problem is a one-dimensional spring-block model describing earthquake faulting. The dynamics of earthquakes is naturally a multiple timescale problem: the timescale of earthquake ruptures is very short, when compared to the time interval between two consecutive ruptures. We identify a small parameter $\varepsilon$ that describes the separation between the timescales, so that $\varepsilon = 0$ idealises the complete timescale separation. Earthquake faulting problems also have multiple spatial scales. The action of friction is generally explained as the loss and restoration of linkages between the surface asperities at the molecular scale. However, the consequences of friction are noticeable at much larger scales, like hundreds of kilometers. By using geometric singular perturbation theory and the blow-up method, we provide a detailed description of the periodicity of the earthquake episodes. In particular, we show that attracting limit cycles arise from a degenerate Hopf bifurcation, whose degeneracy is due to an underlying Hamiltonian structure that leads to large amplitude oscillations. We use a Poincaré compactification to study the system near infinity. At infinity, the critical manifold loses hyperbolicity with an exponential rate. We use an adaptation of the blow-up method to recover the hyperbolicity. This enables the identification of a new attracting manifold, that organises the dynamics at infinity for $\varepsilon = 0$. This in turn leads to the formulation of a conjecture on the behaviour of the limit cycles as the timescale separation increases for $0 < \varepsilon < 1$. We illustrate our findings with numerics, and outline the proof of the conjecture. We also discuss how our results can be used to study a similar class of problems. The second mechanical problem is a friction oscillator subject to stiction. The vector field of this discontinuous model does not follow the Filippov convention, and the concept of Filippov solutions cannot be used. Furthermore, some Carathéodory solutions are unphysical. Therefore, we introduce the concept of stiction solutions: these are the Carathéodory solutions that are physically relevant, i.e. the ones that follow the stiction law. However, we find that some of the stiction solutions are forward non-unique in subregions of the slip onset. We call these solutions singular, in contrast to the regular stiction solutions that are forward unique. In order to further the understanding of the non-unique dynamics, we introduce a regularization of the model. This gives a singularly perturbed problem that captures the main features of the original discontinuous problem. We identify a repelling slow manifold that separates the forward slipping to forward sticking solutions, leading to a high sensitivity to the initial conditions. On this slow manifold we find canard trajectories, that have the physical interpretation of delaying the slip onset. We show numerically that the regularized problem has a family of periodic orbits interacting with the canards. We observe that this family is unstable of saddle type and that it connects, in the rigid body limit, the two regular, slip-stick branches of the discontinuous problem, that were otherwise disconnected.

**General information**

State: Submitted
Organisations: Department of Applied Mathematics and Computer Science, Mathematics
Authors: Bossolini, E. (Intern), Brøns, M. (Intern), Kristiansen, K. U. (Intern)
Singular limit analysis of a model for earthquake faulting

In this paper we consider the one dimensional spring-block model describing earthquake faulting. By using geometric singular perturbation theory and the blow-up method we provide a detailed description of the periodicity of the earthquake episodes. In particular, the limit cycles arise from a degenerate Hopf bifurcation whose degeneracy is due to an underlying Hamiltonian structure that leads to large amplitude oscillations. We use a Poincaré compactification to study the system near infinity. At infinity the critical manifold loses hyperbolicity with an exponential rate. We use an adaptation of the blow-up method to recover the hyperbolicity. This enables the identification of a new attracting manifold that organises the dynamics at infinity. This in turn leads to the formulation of a conjecture on the behaviour of the limit cycles as the time-scale separation increases. We provide the basic foundation for the proof of this conjecture and illustrate our findings with numerics.

General information
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Web of Science (2017): Indexed Yes
Regularization by External Variables

Regularization was a big topic at the 2016 CRM Intensive Research Program on Advances in Nonsmooth Dynamics. There are many open questions concerning well known kinds of regularization (e.g., by smoothing or hysteresis). Here, we propose a framework for an alternative and important kind of regularization, by external variables that shadow either the state or the switch of the original system. The shadow systems are derived from and inspired by various applications in electronic control, predator-prey preference, time delay, and genetic regulation.

General information
State: Published
Organisations: Department of Applied Mathematics and Computer Science, Mathematics, University of Victoria, University of Manchester, University of Bristol
Authors: Bossolini, E. (Intern), Edwards, R. (Ekstern), Glendinning, P. A. (Ekstern), Jeffrey, M. R. (Ekstern), Webber, S. (Ekstern)
Thermal properties of Fiber ropes

There is a trend within the oil and gas market to shift from steel wire ropes to fiber ropes for lifting, hoisting and mooring applications. The cost of fiber ropes is about 2-3 times that of steel wire ropes, but the natural buoyancy of fiber ropes reduces the overall weight resulting in smaller cranes and thereby reduces the overall costs. For heave compensation, a rope is typically of 3-4000 meters long, such that one rope costs in the order of 7.5 million dollars. The current practice on when to discard a fiber rope is through visual inspections done manually with large safety factors. This means that the rope is discarded before it is necessary, increasing the overall life-cycle costs. The offshore industry wants a better monitoring system to understand when the fiber rope must be replaced.

General information

State: Published
Organisations: Department of Applied Mathematics and Computer Science, Dynamical Systems, Center for Intelligent Drug Delivery and Sensing Using Microcontainers and Nanomechanics, Technical University of Denmark, University of Southern Denmark
Authors: Bossolini, E. (Intern), Nielsen, O. W. (Ekstern), Oland, E. (Ekstern), Sørensen, M. P. (Intern), Veje, C. (Ekstern)
Number of pages: 4
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Main Research Area: Technical/natural sciences
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Projects:

From non-smooth to smooth - on regularization using slow-fast theory

Department of Applied Mathematics and Computer Science
Period: 15/10/2014 → 13/12/2017
Number of participants: 7
Phd Student:
Bossolini, Elena (Intern)
Supervisor:
Galvanetto, Ugo (Ekstern)
Kristiansen, Kristian Uldali (Intern)
Main Supervisor:
Brøns, Morten (Intern)
Examiner:
Henriksen, Christian (Intern)
Hogan, Stephen John (Ekstern)
Szmolyan, Peter (Ekstern)

Financing sources

Source: Internal funding (public)
Name of research programme: Institut stipendie (DTU)
Activities:

Canards in Stiction: On Solutions of a Friction Oscillator by Regularization
Period: 22 May 2017
Elena Bossolini (Speaker)
Department of Applied Mathematics and Computer Science
Mathematics

Description
We consider the problem of the friction oscillator using the stiction model of friction. This friction law has a discontinuity between the dynamic and the static regime. The discontinuity set has a sticking region in which the forward solution is non-unique. In particular, there are special points along these segments where the solution is tangent to the boundary of the discontinuity set. In order to resolve this uncertainty, we introduce a regularization of the vector field and we obtain a multiple-time scale problem. Here the special points of the piecewise-smooth problem become folded saddles and a canard solution appears. We study the interaction of periodic orbits with the canard and we find that the the regularized problem has solutions that do not appear in the original problem.
Degree of recognition: International
Links:
http://meetings.siam.org/sess/dsp_programsess.cfm?SESSIONCODE=61861 (Minisymposium description)

Related event

SIAM Conference on Applications of Dynamical Systems 2017
21/05/2017 → 26/05/2017
Snowbird, United States
Activity: Talks and presentations › Conference presentations

SIAM Conference on Applications of Dynamical Systems
Elena Bossolini (Participant)
Department of Applied Mathematics and Computer Science
Mathematics

Description
Slow-fast analysis of Earthquake Faulting

Description of the ongoing research on the model for earthquake faulting with the rate and state friction law.
Degree of recognition: International
Documents:
Poster_SIAMDS2015

Related event

SIAM Conference on Applications of Dynamical Systems
17/05/2015 → 21/05/2015
Snowbird, Utah, United States
Activity: Attending an event › Participating in or organising a conference

From non-smooth to smooth friction models, using regularisation and slow-fast theory
Elena Bossolini (Speaker)
Department of Applied Mathematics and Computer Science
Mathematics

**Description**
Presented the outline of the PhD project and preliminary results

**Documents:**
Poster_DCAMM2015

**Related event**

**DCAMM 15th Internal Symposium**
16/03/2015 → 18/03/2015
Horsens, Denmark
Activity: Talks and presentations › Conference presentations