The impact ofParametrization on Randomized Search Heuristics

In this work we present runtime analyses of randomized search heuristics (RSH) in various settings that are determined by parameters of the problems, the algorithms and also exogenous parameters like noise. In the process we provide new techniques for the theoretical analysis of RSH as well as new optimization algorithms. We consider the following topics.

Escaping local optima using local search. We analyze memetic algorithms, i.e. evolutionary algorithms equipped with a local search after mutation. To this end we consider the (1+1) EA equipped with Standard Local Search (SLS) and Variable-Depth Search (VDS) on an artificial test function. We determine features of the fitness landscape that lead to the (1+1) EA using SLS outperforming the (1+1) EA using VDS with an exponential performance gap. Moreover, we present a new local search operator, Opportunistic Local Search (OLS), that can deal with such features in the landscape and show that the (1+1) EA with OLS can efficiently optimize a discretized Rastrigin function. Stochastic fitness functions. We analyze the role of populations in stochastic optimization. We assume that the objective function is subject to noise, introducing stochastic errors in its evaluation. On classical test functions, such noise makes optimization by the simple (1+1) EA hillclimber infeasible even in exponential time. Interestingly, the use of parent and offspring populations of only logarithmic size turns the algorithm into an efficient one. The results are obtained by drift analysis. An asymptotic expansion of the expected runtime of the (1+λ) EA on ONEMAX. We consider the (1+λ) EA with mutation probability c/n, where c > 0 is a constant on ONEMAX. We give an asymptotic expansion for the expected runtime depending on both c and λ. Our results show that c = 1 is the optimal mutation rate for λ = o(loglogdlogd/logdlogdlogd) and that c only has an impact on the lower-order terms of the expected runtime, i.e. c = 1 is no longer the only optimal mutation rate. Our methods are strongly based on variable drift theorems for upper and lower bounds and a precise analysis of order statistics of the binomial distribution. To the best of our knowledge this is the first tight runtime analysis of a population-based EA, up to lower-order terms. Furthermore, we develop helpful stochastic tools for runtime analyses. Optimal mutation rates for the (1+λ) EA on ONEMAX. We consider the (1+λ) EA with mutation probability c/n on ONEMAX, where c > 0 and λ are constant. We present an improved variable drift theorem that weakens the requirement that no large steps towards the optimum may occur in the process to a stochastic one, reducing the analysis of the expected optimization time to finding an exact expression for the drift. We formalize an exact closed-form expression for the drift and provide small error approximations that are very efficient to compute. Self-adjusting mutation rates for the (1+λ) EA on ONEMAX. We propose a new mechanism to self-adjust the mutation rate in population-based evolutionary algorithms. It consists of creating half the offspring with a higher and the rest with a lower mutation rate. The mutation rate is then adjusted, based on the success of the subpopulations. We show that the (1+λ) EA optimizes ONEMAX in an expected optimization time of O(nλ/logλ + logdlogd) which has been shown to be best-possible among all λ-parallel mutation-based unbiased black-box algorithms.
The (1+λ) evolutionary algorithm with self-adjusting mutation rate
We propose a new way to self-adjust the mutation rate in population-based evolutionary algorithms. Roughly speaking, it consists of creating half the offspring with a mutation rate that is twice the current mutation rate and the other half with half the current rate. The mutation rate is then updated to the rate used in that subpopulation which contains the best offspring. We analyze how the (1 + A) evolutionary algorithm with this self-adjusting mutation rate optimizes the OneMax test function. We prove that this dynamic version of the (1 + A) EA finds the optimum in an expected optimization time (number of fitness evaluations) of O(nA/log A + n log n). This time is asymptotically smaller than the optimization time of the classic (1 + A) EA. Previous work shows that this performance is best-possible among all A-parallel mutation-based unbiased black-box algorithms. This result shows that the new way of adjusting the mutation rate can find optimal dynamic parameter values on the fly. Since our adjustment mechanism is simpler than the ones previously used for adjusting the mutation rate and does not have parameters itself, we are optimistic that it will find other applications.

The Interplay of Population Size and Mutation Probability in the (1+λ) EA on OneMax
The ((Formula presented.)) EA with mutation probability c / n, where (Formula presented.), is studied for the classical OneMax function. Its expected optimization time is analyzed exactly (up to lower order terms) as a function of c and (Formula presented.). It turns out that 1 / n is the only optimal mutation probability if (Formula presented.), which is the cut-off point for linear speed-up. However, if (Formula presented.) is above this cut-off point then the standard mutation probability 1 / n is no longer the only optimal choice. Instead, the expected number of generations is (up to lower order terms) independent of c, irrespectively of it being less than 1 or greater. The theoretical results are obtained by a careful study of order statistics of the binomial distribution and variable drift theorems for upper and lower bounds. Experimental supplements shed light on the optimal mutation probability for small problem sizes.
Optimal mutation rates for the (1+λ) EA on OneMax

We study the (1 + λ) EA with mutation probability c/n, where c > 0 is a constant, on the ONEMAX problem. Using an improved variable drift theorem, we show that upper and lower bounds on the expected runtime of the (1+λ) EA obtained from variable drift theorems are at most apart by a small lower order term if the exact drift is known. This reduces the
analysis of expected optimization time to finding an exact expression for the drift.

We then give an exact closed-form expression for the drift and develop a method to approximate it very efficiently, enabling us to determine approximate optimal mutation rates for the \((1+\lambda)\) EA for various parameter settings of \(c\) and \(\lambda\) and also for moderate sizes of \(n\). This makes the need for potentially lengthy and costly experiments in order to optimize the parameters unnecessary.

Interestingly, even for moderate \(n\) and not too small \(\lambda\) it turns out that mutation rates up to 10\% larger than the asymptotically optimal rate \(1/n\) minimize the expected runtime. However, in absolute terms the expected runtime does not change by much when replacing \(1/n\) with the optimal mutation rate.

Robustness of Populations in Stochastic Environments
We consider stochastic versions of OneMax and LeadingOnes and analyze the performance of evolutionary algorithms with and without populations on these problems. It is known that the \((1+1)\) EA on OneMax performs well in the presence of very small noise, but poorly for higher noise levels. We extend these results to LeadingOnes and to many different noise models, showing how the application of drift theory can significantly simplify and generalize previous analyses. Most surprisingly, even small populations (of size \(\Theta(\log n)\)) can make evolutionary algorithms perform well for high noise levels, well outside the abilities of the \((1+1)\) EA. Larger population sizes are even more beneficial; we consider both parent and offspring populations. In this sense, populations are robust in these stochastic settings.
The (1+1) EA with mutation probability $c/n$, where $c>0$ is an arbitrary constant, is studied for the classical OneMax function. Its expected optimization time is analyzed exactly (up to lower order terms) as a function of $c$ and $\lambda$. It turns out that $1/n$ is the only optimal mutation probability if $\lambda=O((\ln n \ln \ln n)/\ln \ln \ln n)$, which is the cut-off point for linear speed-up. However, if $\lambda$ is above this cut-off point then the standard mutation probability $1/n$ is no longer the only optimal choice. Instead, the expected number of generations is (up to lower order terms) independent of $c$, irrespectively of it being less than 1 or greater. The results are obtained by a careful study of order statistics of the binomial distribution and variable drift theorems for upper and lower bounds.

**Population Size vs. Mutation Strength for the (1+\lambda) EA on OneMax**

The (1+\lambda) EA with mutation probability $c/n$, where $c>0$ is an arbitrary constant, is studied for the classical OneMax function. Its expected optimization time is analyzed exactly (up to lower order terms) as a function of $c$ and $\lambda$. It turns out that $1/n$ is the only optimal mutation probability if $\lambda=O((\ln n \ln \ln n)/\ln \ln \ln n)$, which is the cut-off point for linear speed-up. However, if $\lambda$ is above this cut-off point then the standard mutation probability $1/n$ is no longer the only optimal choice. Instead, the expected number of generations is (up to lower order terms) independent of $c$, irrespectively of it being less than 1 or greater. The results are obtained by a careful study of order statistics of the binomial distribution and variable drift theorems for upper and lower bounds.
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Hybridizing Evolutionary Algorithms with Opportunistic Local Search

There is empirical evidence that memetic algorithms (MAs) can outperform plain evolutionary algorithms (EAs). Recently the first runtime analyses have been presented proving the aforementioned conjecture rigorously by investigating Variable-Depth Search, VDS for short (Sudholt, 2008). Sudholt raised the question if there are problems where VDS performs badly. We answer this question in the affirmative in the following way. We analyze MAs with VDS, which is also known as Kernighan-Lin for the TSP, on an artificial problem and show that MAs with a simple first-improvement local search outperform VDS. Moreover, we show that the performance gap is exponential. We analyze the features leading to a failure of VDS and derive a new local search operator, coined Opportunistic Local Search, that can easily overcome regions of the search space where local optima are clustered. The power of this new operator is demonstrated on the Rastrigin function encoded for binary hypercubes. Our results provide further insight into the problem of how to prevent local search algorithms to get stuck in local optima from a theoretical perspective. The methods stem from discrete probability theory and combinatorics.
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