Classification of complex polynomial vector fields in one complex variable

This paper classifies the global structure of monic and centred one-variable complex polynomial vector fields. The classification is achieved by means of combinatorial and analytic data. More specifically, given a polynomial vector field, we construct a combinatorial invariant, describing the topology, and a set of analytic invariants, describing the geometry. Conversely, given admissible combinatorial and analytic data sets, we show using surgery the existence of a unique monic and centred polynomial vector field realizing the given invariants. This is the content of the Structure Theorem, the main result of the paper. This result is an extension and refinement of Douady et al. (Champs de vecteurs polynomiaux sur C. Unpublished manuscript) classification of the structurally stable polynomial vector fields. We further review some general concepts for completeness and show that vector fields in the same combinatorial class have flows that are quasi-conformally equivalent.

Publication information
Journal: Journal of Difference Equations and Applications
Volume: 16
Issue number: 5-6
ISSN (Print): 1023-6198
Ratings:
BFI (2018): BFI-level 1
BFI (2017): BFI-level 1
Web of Science (2017): Indexed Yes
Scopus rating (2016): SJR 0.813 SNIP 1.02 CiteScore 0.84
BFI (2015): BFI-level 1
Scopus rating (2015): SJR 0.826 SNIP 0.947 CiteScore 0.85
BFI (2014): BFI-level 1
Scopus rating (2014): SJR 0.792 SNIP 0.926 CiteScore 0.81
BFI (2013): BFI-level 1
Scopus rating (2013): SJR 0.738 SNIP 1.121 CiteScore 0.9
ISI indexed (2013): ISI indexed yes
BFI (2012): BFI-level 1
Scopus rating (2012): SJR 0.804 SNIP 1.02 CiteScore 0.92
ISI indexed (2012): ISI indexed yes
BFI (2011): BFI-level 1
Scopus rating (2011): SJR 1.254 SNIP 1.199 CiteScore 0.91
ISI indexed (2011): ISI indexed yes
The objective of the Ph.D. project is to initiate a classification of bifurcations of meromorphic vector fields and to clarify their relation to circle packings. Technological applications are to image analysis and to effective grid generation using discrete conformal mappings. The two branches of dynamical systems, continuous and discrete, correspond to the study of differential equations (vector fields) and iteration of mappings respectively. In holomorphic dynamics, the systems studied are restricted to those described by holomorphic (complex analytic) functions or meromorphic (allowing poles as singularities) functions. There already exists a well-developed theory for iterative holomorphic dynamical systems, and successful relations found between iteration theory and flows of vector fields have been one of the main motivations for the recent interest in holomorphic vector fields. Restricting to structurally stable vector fields, there is an underlying dynamically defined triangulation of the plane. Circle packings are a means to realize such a given combinatorial structure.

About 20 years ago, W. Thurston suggested applying circle packings to obtain approximations to Riemann mappings. This gave rise to the development of a theory of discrete analytic functions, which is a new tool in conformal geometry that can be used to implement many of the classical tools from complex analysis. Circle packing is a relatively new subject that has a great potential for technological applications, specifically for imaging problems. Since the class of complex polynomial vector fields in the plane is natural to consider, it is remarkable that its study has only begun very recently. There are numerous fundamental questions that are still open, both in the general classification of these vector fields, the decomposition of parameter spaces into structurally stable domains, and a description of the bifurcations. The same holds true for questions related to vector fields on the Riemann sphere and Riemann surfaces of higher genus. The overall objectives of this Ph.D.-study are to characterize the decomposition of parameter spaces of meromorphic vector fields on Riemann surfaces of low genus and a description of the bifurcations and to implement characteristic vector fields on such surfaces using circle packings. Furthermore, when the implementations using circle packings have been established, applications in conformal geometry and image analysis will be investigated in collaboration with faculty members at MAT and IMM.
Polynomial Vector Fields in One Complex Variable

In recent years Adrien Douady was interested in polynomial vector fields, both in relation to iteration theory and as a topic on their own. This talk is based on his work with Pierrette Sentenac, work of Xavier Buff and Tan Lei, and my own joint work with Kealey Dias.

Pas de science sans passion

Pas de science sans passion
MINICOURSE Tableaus and their use in Holomorphic Dynamics. TITLE of class I: Puzzles and para-puzzles, and the divergence property. ABSTRACT: The geometrical part: Puzzles in the most fundamental cases, i.e. associated with a polynomial, which either belongs to the Yoccoz-class of quadratic polynomials or to the bounded/unbounded-class of cubic polynomials (with one bounded and one unbounded critical orbit). Para-puzzles in the quadratic case. The analytical part: The divergence property and its usefulness. - An infinite set of open disjoint annuli embedded in a bounded open annuli, all of the same homotopy type, is said to have the divergence property, if the infinite series of moduli of these annuli is divergent. TITLE of class II: Tableaus. ABSTRACT: The combinatorial part: Tableaus associated with a polynomial belonging to the Yoccoz-class or the bounded/unbounded class. The tableau rules, classification of critical tableaus, the Fibonacci critical tableau. TITLE of class III: Points are points. ABSTRACT: Combining the geometrical, analytical and combinatorial parts to conclude either local connectivity of the Julia set of a polynomial in the Yoccoz-class or total disconnectivity of the Julia set of a polynomial in the bounded/unbounded class (i.e. the Julia set is a Cantor set). In both cases one proves that (certain) connected components are reduced to point components. Therefore, Adrien Douady liked to say that one proves that "points are points".

General information
State: Published
Organisations: Department of Mathematics
Authors: Branner, B. (Intern)
Publication date: 2007
Event: Abstract from Symbolic Dynamics in Complex Dynamics, Søminestationen, Holbæk, Denmark,
Main Research Area: Technical/natural sciences
Source: orbit
Source-ID: 207464
Publication: Research › Conference abstract for conference – Annual report year: 2007

Adrien Douady (25 september 1935 - 2 november 2006)

General information
State: Published
Organisations: Department of Mathematics, Dynamical systems, Roskilde Universitet
Authors: Branner, B. (Intern), Petersen, C. L. (Ekstern), Henriksen, C. (Intern)
Pages: 23-25
Publication date: 2006
Main Research Area: Technical/natural sciences

Publication information
Journal: Matilde
Issue number: 29
ISSN (Print): 1399-5901
Ratings:
ISI indexed (2013): ISI indexed no
ISI indexed (2012): ISI indexed no
ISI indexed (2011): ISI indexed no
Original language: Danish
Source: orbit
Source-ID: 194092
Publication: Communication › Journal article – Annual report year: 2006

A Holomorphic Tale
Glimpses of Holomorphic Dynamics in light of John Milnor's influence on its development.

General information
State: Published
Organisations: Department of Mathematics
Authors: Branner, B. (Intern)
Publication date: 2006
Event: Abstract from Holomorphic Dynamics Workshop : In celebration of John Milnor 75th birthday, The Fields Institute, Toronto, Canada,
Main Research Area: Technical/natural sciences
Source: orbit
Source-ID: 194088
Publication: Research - peer-review › Conference abstract for conference – Annual report year: 2006
Topological properties of dynamically defined sets in holomorphic dynamics of one variable

General information
State: Published
Organisations: Department of Mathematics
Authors: Branner, B. (Intern)
Publication date: 2005
Event:
Main Research Area: Technical/natural sciences
Source: orbit
Source-ID: 185552
Publication: Research › Conference abstract for conference – Annual report year: 2005

Geometry of honeybee combs

General information
State: Published
Organisations: Dynamical systems, Department of Mathematics
Authors: Branner, B. (Intern)
Publication date: 2004
Event: Poster session presented at 10th International Congress on Mathematical Education, Copenhagen, Denmark.
Main Research Area: Technical/natural sciences
Source: orbit
Source-ID: 154453
Publication: Research - peer-review › Poster – Annual report year: 2004

Interview with Vagn Lundsgaard Hansen

General information
State: Published
Organisations: Department of Mathematics
Authors: Branner, B. (Intern), Markvorsen, S. (Intern)
Pages: 15-19
Publication date: 2003
Main Research Area: Technical/natural sciences

Publication information
Journal: Newsletter of the European Mathematical Society
Volume: 47
ISSN (Print): 1027-488X
Ratings:
ISI indexed (2013): ISI indexed no
ISI indexed (2012): ISI indexed no
ISI indexed (2011): ISI indexed no
Original language: English
Source: orbit
Source-ID: 185383
Publication: Communication › Journal article – Annual report year: 2003

On the foundation of Mathematica Scandinavica

General information
State: Published
Organisations: Department of Mathematics
Authors: Branner, B. (Intern)
Pages: 5-19
Publication date: 2003
Main Research Area: Technical/natural sciences

Publication information
Journal: MATH SCAND
Volume: 93
Caspar Wessel (1745-1818). Surveyor and Mathematician.

General information
State: Published
Organisations: Department of Mathematics, University of Oslo
Authors: Branner, B. (Intern), Johansen, N. V. (Ekstern)
Publication date: 1999

Host publication information
Title of host publication: Caspar Wessel. On the Analytical Representation of Direction.
Place of publication: Copenhagen
Publisher: The Royal Danish Academy of Sciences and Letters
Main Research Area: Technical/natural sciences
Conference: Wessel Symposium, the Royal Danish Academy of Sciences and Letters, Copenhagen, 01/01/1998
Source: orbit
Caspar Wessel on representing complex numbers (1799)

General information
State: Published
Organisations: Department of Mathematics
Authors: Branner, B. (Intern)
Pages: 13-16
Publication date: 1999
Main Research Area: Technical/natural sciences

Publication information
Journal: Newsletter of European Mathematical Society
Issue number: 33
Original language: English
Source: orbit
Source-ID: 174687
Publication: Research › Journal article – Annual report year: 1999

Caspar Wessel. On the Analytical Representation of Direction: An Attempt Applied Chiefly to Solving Plane and Spherical Polygons

General information
State: Published
Organisations: Department of Mathematics
Authors: Branner, B. (ed.) (Intern)
Publication date: 1999
Main Research Area: Technical/natural sciences

Publication information
Place of publication: Copenhagen
Publisher: The Royal Danish Academy of Sciences and Letters
Original language: English
Main Research Area: Technical/natural sciences
Source: orbit
Source-ID: 174684
Publication: Research - peer-review › Book – Annual report year: 1999

Homeomorphisms Between Limbs of the Mandelbrot Set

General information
State: Published
Organisations: Department of Mathematics, Universitat de Barcelona
Authors: Branner, B. (Intern), Fagella, N. (Ekstern)
Pages: 327-390
Publication date: 1999
Main Research Area: Technical/natural sciences

Publication information
Journal: Journal of Geometric Analysis
Volume: 9
Issue number: 3
ISSN (Print): 1050-6926
Ratings:
BFI (2018): BFI-level 2
BFI (2017): BFI-level 2
Web of Science (2017): Indexed Yes
BFI (2016): BFI-level 2
Scopus rating (2016): CiteScore 0.95 SJR 1.537 SNIP 1.211
BFI (2015): BFI-level 2
Scopus rating (2015): SJR 1.57 SNIP 1.49 CiteScore 0.94
An Interview of Professor Friedrich Hirzebruch

General information
State: Published
Organisations: Department of Mathematics
Authors: Branner, B. (Intern)
Pages: 4-7
Publication date: 1998
Main Research Area: Technical/natural sciences

Publication information
Journal: Newsletter of European Mathematical Society
Volume: Sep
Issue number: 29
Original language: English
Source: orbit
Source-ID: 170103
Publication: Research › Journal article – Annual report year: 1998

Holomorphic dynamical systems in the complex plane
Holomorphic Dynamical Systems in the Complex Plane

General information
State: Published
Organisations: Department of Mathematics
Authors: Branner, B. (Intern)
Number of pages: 20
Publication date: 1996

Publication information
Original language: English
Main Research Area: Technical/natural sciences
Source: orbit
Source-ID: 165763
Publication: Research - peer-review › Report – Annual report year: 1996

Proceedings of the seventh EWM meeting, European Women in Mathematics

General information
State: Published
Organisations: Department of Mathematics
Authors: Branner, B. (ed.) (Intern), Flagella, N. (ed.) (Ekstern)
Number of pages: 234
Publication date: 1996

Publication information
Real and Complex Dynamical Systems

Classification of Meromorphic Vector Fields

A complex polynomial defines a holomorphic vector field in the complex plane. The quasi-conformal conjugacy class of the polynomial is completely determined by a combinatorial invariant. Furthermore, within each combinatorial class the polynomial is uniquely determined by a finite number (settled by the combinatorial class) of complex numbers. This fundamental classification of complex polynomial vector fields is proved using surgery. Further developments are to classify possible bifurcations, to understand the decomposition of parameter spaces due to the different combinatorial classes and the bifurcations among them, and also to extend to meromorphic vector fields arising from rational functions on the Riemann sphere.

Dias, Kealey (Intern)
Henriksen, Christian (Intern)
Project Manager, organisational:
Branner, Bodil (Intern)

Financing sources
Source: Ph.d Central finansieret
Name of research programme: Ph.d Central finansieret
Amount: 115,000.00 Danish Kroner

Meromorphic Vector Fields and Circle Packings

Department of Mathematics
Period: 01/09/2006 → 10/02/2010
Number of participants: 6
Phd Student:
Dias, Kealey (Intern)
Supervisor:
Branner, Bodil (Intern)
Main Supervisor:
Henriksen, Christian (Intern)
Examiner:
Markvorsen, Steen (Intern)
Epstein, Adam Lawrence (Ekstern)
Petersen, Carsten Lunde (Intern)

Financing sources
Source: Internal funding (public)
Name of research programme: DTU-lønnet stipendie
Project: PhD

Limes

Department of Mathematics
Period: 21/06/2000 → 31/03/2004
Number of participants: 1
Project participant:
Branner, Bodil (Intern)

Financing sources
Source: Forsk. EU - Rammeprogram
Name of research programme: Forsk. EU - Rammeprogram
Amount: 586,751.00 Danish Kroner
Project: Holomorf dynamik og Herman-ringe

Holomorphic Dynamics, Herman Rings.
Geometry of (possibly) degenerated Herman Rings. Geometry of the set of points in parameter space corresponding to maps with a Herman ring exhibiting certain characteristics such as rotation number, modulus and wringing.

Department of Mathematics
Period: 01/09/1997 → 30/03/2001
Number of participants: 6
Phd Student:
Henriksen, Christian (Intern)
Supervisor:
Petersen, Carsten Lunde (Intern)
Main Supervisor:
Branner, Bodil (Intern)
Examiner:
Hjorth, Poul G. (Intern)
Astala, Kari (Ekstern)
Douady, Adrien (Ekstern)

Financing sources
Source: Internal funding (public)
Name of research programme: DTU-Su Stipendium, Eksperiment
Project: PhD

Holomorf dynamik og Herman-ringe

Department of Mathematics
Period: 01/09/1997 → 30/03/2001
Number of participants: 6
Phd Student:
Henriksen, Christian (Intern)
Supervisor:
Petersen, Carsten Lunde (Intern)
Main Supervisor:
Branner, Bodil (Intern)
Examiner:
Hjorth, Poul G. (Intern)
Astala, Kari (Ekstern)
Douady, Adrien (Ekstern)

Financing sources
Source: Internal funding (public)
Name of research programme: DTU-Su Stipendium, Eksperiment
Project: PhD
Mathematical Study and Visualization of Complex Dynamical Systems
The aim of this project is to visualize hardly accessible phenomena within complex dynamical systems, such as: the parabolical implosion, holomorphic surgery and renormalization, as well as to illustrate the elementary phenomena: iteration, topological models and potential theoretical tools. In order to be able to visualize the phenomena mentioned, thorough mathematical studies are necessary. A number of known proofs need to be modified and new proofs established in order to achieve the mathematical visualization. The visualization media is the video, but it is accompanied with explicationary material, interactive programs (possibly as CD-ROM) and finally classical mathematical publications. The receiving audience includes mathematical high-school students to fellow researchers within the area. Keywords: Holomorphic dynamics, algorithms, computer animation.

Department of Mathematics
Universite Paris-Sud
Atelier EcoutezVoir
Period: 01/09/1996 → 31/08/1997
Number of participants: 4
Project participant:
Branner, Bodil (Intern)
Douady, Adrien (Ekstern)
Tisseur, Francois (Ekstern)
Project Manager, organisational:
Sørensen, Dan Erik Krarup (Intern)

Financing sources
Source: Unknown
Name of research programme: Ukendt
Amount: 397,489.00 Danish Kroner
Project

Non-locally Connected Quadratic Julia Sets
Work is done with non-linear dynamical systems, which appears from iterating with a complex, quadratic polynomial. The main purpose is to increase the understanding of the dynamic, geometry and topology for polynomials with locally connected Julia set. As an example, the behaviour of certain external rays under repeated parabolic pertubations are investigated. As the most important side results we should mentioned the achievement of the first known examples of connected, but not curve-connected Julia sets, as well as non-robust, renormalizeable (infinitely many times) quadratic polynomials. Present work is considering the possibility to relate the polynomial classes found to the so-called Diophantic conditions in the theory of numbers and the possibility to find quadratic Julia sets with positive measure. Keywords: External rays, local connectedness, robustness and Julia sets.

Department of Mathematics
Universite Paris-Sud
Cornell University
Roskilde Universitet
Period: 01/09/1996 → 31/08/1997
Number of participants: 6
Project participant:
Branner, Bodil (Intern)
Willumset, Pia B.N. (Intern)
Douady, Adrien (Ekstern)
Hubbard, John H. (Ekstern)
Petersen, Carsten Lunde (Ekstern)
Project Manager, organisational:
Sørensen, Dan Erik Krarup (Intern)

Financing sources
Source: Unknown
Name of research programme: Ukendt
Amount: 397,489.00 Danish Kroner
Project
Holomorphic Dynamical Systems

In holomorphic dynamics one studies iteration of holomorphic maps, in particular polynomials. The dynamical space (typically the complex plane or the Riemann sphere) is divided into the Fatou set, where the dynamics is stable, and the Julia set where the dynamics is chaotic. The goal is not only to understand the topology and geometry of Julia sets of individual holomorphic maps, but also to understand how the Julia set and the dynamics vary with the map, in particular to understand bifurcation sets of maps where the dynamics change qualitatively. The scope of the project is broad. It concentrates on describing special types of results and techniques, namely those for which a transfer of results is possible from dynamical spaces to parameter spaces, parametrizing families of holomorphic maps. For instance, results obtained using puzzles in dynamical spaces and para-puzzles in parameter spaces as a tool.

Department of Mathematics

University of Paris-Sud - University of Paris XI
Cornell University
University of Warwick
Universidad Autonoma de Barcelona

Period: 01/01/1996 → …
Number of participants: 5
Project participant:
Douady, Adrien (Ekstern)
Hubbard, John H. (Ekstern)
Lei, Tan (Ekstern)
Fagella, Nuria (Ekstern)

Branner, Bodil (Intern)

Financing sources
Source: Unknown
Name of research programme: Ukendt
Amount: 20,000.00 Danish Kroner
Project

Caspar Wessel and the Complex Numbers

In 1797 a paper by the Norwegian surveyor Caspar Wessel was presented to the Royal Danish Academy of Sciences and Letters. In this paper he introduced the geometrical representation of complex numbers. From his surveying reports we know that he had the idea at least as early as 1787. A biography relating his mathematical paper to his work as surveyor under the auspices of the Academy will appear in a book, edited by Jesper Lutzen, and published by the Academy in 1998.

Department of Mathematics

University of Oslo
Period: 01/06/1995 → …
Number of participants: 2
Project participant:
Johansen, Nils Voje (Ekstern)

Branner, Bodil (Intern)

Quasi-conformal Surgery

The technique of quasi-conformal surgery in holomorphic dynamics was initiated by Sullivan, Douady, Hubbard and Shishikura in the early eighties. The method is to create new dynamical systems out of some given ones, by changing not only the dynamical plane (through cutting and sewing) and the map defining the dynamical system, but also the complex structure of the new dynamical plane. The theory of quasi-conformal mappings is the basic tool. Surgery techniques appear to be particularly successful, when two families of maps can be related in such a way that dynamical similarities are transferred to similarities between structures in the corresponding parameter spaces. Homeomorphisms between p/q-limbs of the Mandelbrot set, with fixed denominator q, have been obtained. Generalizations are currently being investigated.

Department of Mathematics
Fagella, Nuria (Ekstern)
Project Manager, organisational:
Branner, Bodil (Intern)

Kleinske grupper og Holomorfe dynamiske systemer
Department of Mathematics
Period: 01/02/1993 → 24/07/1997
Number of participants: 2
Phd Student:
Willumsen, Pia B.N. (Intern)
Main Supervisor:
Branner, Bodil (Intern)

Financing sources
Source: Internal funding (public)
Name of research programme: Forskningsrådsstip.-SU, Eksp
Project: PhD

Komplekse dynamiske systemer.
Department of Mathematics
Period: 01/03/1992 → 16/12/1994
Number of participants: 2
Phd Student:
Sørensen, Dan Erik Krarup (Intern)
Main Supervisor:
Branner, Bodil (Intern)

Financing sources
Source: Internal funding (public)
Name of research programme: DTU-stipendium
Project: PhD