Formalization of the Resolution Calculus for First-Order Logic

I present a formalization in Isabelle/HOL of the resolution calculus for first-order logic with formal soundness and completeness proofs. To prove the calculus sound, I use the substitution lemma, and to prove it complete, I use Herbrand interpretations and semantic trees. The correspondence between unsatisfiable sets of clauses and finite semantic trees is formalized in Herbrand's theorem. I discuss the difficulties that I had formalizing proofs of the lifting lemma found in the literature, and I formalize a correct proof. The completeness proof is by induction on the size of a finite semantic tree. Throughout the paper I emphasize details that are often glossed over in paper proofs. I give a thorough overview of formalizations of first-order logic found in the literature. The formalization of resolution is part of the IsaFoL project, which is an effort to formalize logics in Isabelle/HOL.

First-Order Logic According to Harrison

We present a certified declarative first-order prover with equality based on John Harrison’s Handbook of Practical Logic and Automated Reasoning, Cambridge University Press, 2009. ML code reflection is used such that the entire prover can be executed within Isabelle as a very simple interactive proof assistant. As examples we consider Pelletier’s problems 1-46.
Formalization of Many-Valued Logics
Partiality is a key challenge for computational approaches to artificial intelligence in general and natural language in particular. Various extensions of classical two-valued logic to many-valued logics have been investigated in order to meet this challenge. We use the proof assistant Isabelle to formalize the syntax and semantics of many-valued logics with determinate as well as indeterminate truth values. The formalization allows for a concise presentation and makes automated verification possible.

Formalizing a Paraconsistent Logic in the Isabelle Proof Assistant
We present a formalization of a so-called paraconsistent logic that avoids the catastrophic explosiveness of inconsistency in classical logic. The paraconsistent logic has a countably infinite number of non-classical truth values. We show how to use the proof assistant Isabelle to formally prove theorems in the logic as well as meta-theorems about the logic. In particular, we formalize a meta-theorem that allows us to reduce the infinite number of truth values to a finite number of truth values, for a given formula, and we use this result in a formalization of a small case study.
We present a new software tool for teaching logic based on natural deduction. Its proof system is formalized in the proof assistant Isabelle such that its definition is very precise. Soundness of the formalization has been proved in Isabelle. The tool is open source software developed in TypeScript / JavaScript and can thus be used directly in a browser without any further installation. Although developed for computer science bachelor students who are used to study and program concrete computer code in a programming language we consider the approach relevant for a broader audience and for other proof systems as well.

**NaDeA: A Natural Deduction Assistant with a Formalization in Isabelle**

We present a new software tool for teaching logic based on natural deduction. Its proof system is formalized in the proof assistant Isabelle such that its definition is very precise. Soundness of the formalization has been proved in Isabelle. The tool is open source software developed in TypeScript / JavaScript and can thus be used directly in a browser without any further installation. Although developed for computer science bachelor students who are used to study and program concrete computer code in a programming language we consider the approach relevant for a broader audience and for other proof systems as well.

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**Code Generation for a Simple First-Order Prover**

We present Standard ML code generation in Isabelle/HOL of a sound and complete prover for first-order logic, taking formalizations by Tom Ridge and others as the starting point. We also define a set of so-called unfolding rules and show how to use these as a simple prover, with the aim of using the approach for teaching logic and verification to computer science students at the bachelor level.

**General information**

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Formalization of the Resolution Calculus for First-Order Logic

A formalization in Isabelle/HOL of the resolution calculus for first-order logic is presented. Its soundness and completeness are formally proven using the substitution lemma, semantic trees, Herbrand’s theorem, and the lifting lemma. In contrast to previous formalizations of resolution, it considers first-order logic with full first-order terms, instead of the propositional case.

How I convinced Isabelle that resolution is complete

Isabelle is a proof assistant, i.e., a computer program that can help its user conduct proofs and check their correctness. In this talk I motivate the use of proof assistants, and I explain how I used Isabelle to prove a logical system, the resolution calculus, sound and complete.
Paraconsistency
Paraconsistency is about handling inconsistency in a coherent way. In classical and intuitionistic logic everything follows from an inconsistent theory. A paraconsistent logic avoids the explosion. Quite a few applications in computer science and engineering are discussed in the Intelligent Systems Reference Library Volume 110: Towards Paraconsistent Engineering (Springer 2016). We formalize a paraconsistent many-valued logic that we motivated and described in a special issue on logical approaches to paraconsistency (Journal of Applied Non-Classical Logics 2005). We limit ourselves to the propositional fragment of the higher-order logic. The logic is based on so-called key equalities and has a countably infinite number of truth values. We prove theorems in the logic using the definition of validity. We verify truth tables and also counterexamples for non-theorems. We prove meta-theorems about the logic and finally we investigate a case study.

The Resolution Calculus for First-Order Logic
This theory is a formalization of the resolution calculus for first-order logic. It is proven sound and complete. The soundness proof uses the substitution lemma, which shows a correspondence between substitutions and updates to an environment. The completeness proof uses semantic trees, i.e. trees whose paths are partial Herbrand interpretations. It employs Herbrand's theorem in a formulation which states that an unsatisfiable set of clauses has a finite closed semantic tree. It also uses the lifting lemma which lifts resolution derivation steps from the ground world up to the first-order world. The theory is presented in a paper at the International Conference on Interactive Theorem Proving [7] and an earlier version in an MSc thesis [6]. It mostly follows textbooks by Ben-Ari [1], Chang and Lee [3], and Leitsch [4]. The theory is part of the IsaFoL project [2].
Verification of an LCF-Style First-Order Prover with Equality

We formalize in Isabelle/HOL the kernel of an LCF-style prover for first-order logic with equality from John Harrison's Handbook of Practical Logic and Automated Reasoning. We prove the kernel sound and generate Standard ML code from the formalization. The generated code can then serve as a verified kernel. By doing this we also obtain verified components such as derived rules, a tableau prover, tactics, and a small declarative interactive theorem prover. We test that the kernel and the components give the same results as Harrison's original on all the examples from his book. The formalization is 600 lines and is available online.

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Formalization of Algorithms and Logical Inference Systems in Proof Assistants

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Meta-Logical Reasoning in Higher-Order Logic
The semantics of first-order logic (FOL) can be described in the meta-language of higher-order logic (HOL). Using HOL one can prove key properties of FOL such as soundness and completeness. Furthermore, one can prove sentences in FOL valid using the formalized FOL semantics. To aid in the construction of the proof an interactive proof assistant like Isabelle can be used. The proof assistant can even automate simple proofs using the formalized FOL semantics.

NaDeA: A Natural Deduction Assistant with a Formalization in Isabelle
We present a new software tool for teaching logic based on natural deduction. Its proof system is formalized in the proof assistant Isabelle such that its definition is very precise. Soundness of the formalization has been proved in Isabelle. The tool is open source software developed in TypeScript / JavaScript and can thus be used directly in a browser without any further installation. Although developed for undergraduate computer science students who are used to study and program concrete computer code in a programming language we consider the approach relevant for a broader audience and for other proof systems as well.

Nature-Inspired and Energy Efficient Route Planning
Cars are responsible for substantial CO2 emission worldwide. Computers can help solve this problem by computing shortest routes on maps. A good example of this is the popular Google Maps service. However, such services often require the order of the stops on the route to be fixed. By not enforcing an order on the stops, the route can be made shorter. When, for instance, a furniture dealer has to deliver goods, the order of visiting the customers will often be unimportant. We present a prototype app that can make shorter and more energy efficient routes by allowing it to change the order of the stops. The app is aimed at private persons and small businesses. The app works by using a nature-inspired algorithm called Ant Colony Optimization.
Organisations: Department of Applied Mathematics and Computer Science, Algorithms and Logic, Technical University of Denmark
Authors: Schlichtkrull, A. (Intern), Christensen, J. B. S. (Ekstern), Feld, T. (Ekstern), Hansen, T. B. (Ekstern)
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Formalization of Algorithms and Logical Inference Systems in Proof Assistants
Technical University of Denmark
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Number of participants: 4
Phd Student:
Schlichtkrull, Anders (Intern)
Supervisor:
Blanchette, Jasmin Christian (Ekstern)
Bolander, Thomas (Intern)
Main Supervisor:
Villadsen, Jørgen (Intern)

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Name of research programme: Institut stipendie (DTU)
Project: PhD

Activities:

Proof Assistants and Related Tools - The PART & PART 2 Projects 2017
Period: 9 Nov 2017
Anders Schlichtkrull (Participant)
Department of Applied Mathematics and Computer Science
Algorithms and Logic

Description
IsaFoL - Isabelle Formalization of Logic - A Brief Overview
Talk "IsaFoL - Isabelle Formalization of Logic - A Brief Overview"

Related event

Proof Assistants and Related Tools - The PART & PART 2 Projects 2017
07/09/2017 → …
Kgs. Lyngby, Denmark
Activity: Attending an event › Participating in or organising workshops, courses, seminars etc.

TCS and PAM seminar
Period: 3 Nov 2017
Anders Schlichtkrull (Participant)
Department of Applied Mathematics and Computer Science
Algorithms and Logic

Description
Formalization of an Ordered Resolution Prover in Isabelle/HOL

Abstract:
This is joint work with Jasmin Christian Blanchette, Dmitriy Traytel and Uwe Waldmann. We present a formalization of the first half of Bachmair and Ganzinger's chapter on resolution theorem proving in Isabelle/HOL, culminating with a refutationally complete first-order prover based on ordered resolution with literal selection. We develop general infrastructure and methodology that can form the basis of completeness proofs for related calculi (e.g., superposition). Our work clarifies several fine points in the chapter's text, emphasizing the value of formal proofs in the field of automated reasoning.

Talk at the TCS and PAM seminar of the Theoretical Computer Science group at the Vrije Universiteit Amsterdam.

Related event

Proof Assistants and Related Tools - The PART & PART 2 Projects 2017
Period: 7 Sep 2017
Anders Schlichtkrull (Participant)
Department of Applied Mathematics and Computer Science
Algorithms and Logic

Description
Anders Schlichtkrull (joint work with Jasmin Christian Blanchette, Dmitriy Traytel and Uwe Waldmann): Formalization of an Ordered Resolution Prover in Isabelle/HOL

Talk "Formalization of an Ordered Resolution Prover in Isabelle/HOL" at PART

Related event

Proof Assistants and Related Tools - The PART & PART 2 Projects 2017
07/09/2017 → …
Kgs. Lyngby, Denmark
Activity: Attending an event › Participating in or organising workshops, courses, seminars etc.

MAS & HOL
Period: 28 Sep 2016
Anders Schlichtkrull (Participant)
Department of Applied Mathematics and Computer Science
Algorithms and Logic

Description
Formalization of Algorithms and Logical inference Systems in Proof Assistants

Talk "Formalization of Algorithms and Logical inference Systems in Proof Assistants"

Related event
Formalization of the Resolution Calculus for First-Order Logic

Speaker: Anders Schlichtkrull

Abstract: A formalization in Isabelle/HOL of the resolution calculus for first-order logic is presented. Its soundness and completeness are formally proven using the substitution lemma, semantic trees, Herbrand's theorem, and the lifting lemma. In contrast to previous formalizations of resolution, it considers first-order logic with full first-order terms, instead of the propositional case.

Talk "Formalization of the Resolution Calculus for First-Order Logic" at Club2 of the Chair for Logic and Verification at the Technical University of Munich

Related event

Proof Assistants and Related Tools - The PART & PART 2 Projects 2016

Talk "Computer-Checked Logical Inference Systems" at PART

Related event

PART

Talk "Formalization of Resolution Calculus in Isabelle" at PART

Related event